Hi to you all...read this short reading and see if can get the gist of a **counter example**.

Achievement Indicator:

NL1.4 *Provide and explain a counterexample to disprove a given conjecture.*

When students make a conjecture, they will attempt to either prove their conjecture by constructing a valid argument or they may disprove their conjecture by finding a counterexample. One counterexample is sufficient to disprove a conjecture.

To introduce this topic, students could be asked to come up with counterexamples for statements such as the following:

(i) Any piece of furniture having four legs is a table.

(ii) If something has four wheels, it is a car.

(iii) If the grass is wet, it is raining.

(iv) All students like sports.

When looking for a counterexample, students should continue to use strategies such as drawing pictures or using measuring instruments. After a counterexample is found, they should revise their conjecture to include the new evidence.

When testing conjectures involving numbers, students may be unable to find a counterexample because they often use the same type of number. For example, students may only consider natural numbers. They should be encouraged to try various types of numbers, such as whole numbers and fractions, positive numbers, negative numbers and zero.

It is important for students to realize that if they cannot find a counterexample this does not prove the conjecture.

So if someone makes a conjecture that "all professional NBA basketball players are tall" then you can disprove this statement by providing one and only one counterexample. The counterexample that proves the conjecture false is enough and stands on its own to provide proof the statement is actually incorrect and not true in all cases. Two counterexamples in this case would be Isiah Thomas or Canada's own, Steve Nash. Google them if you want to see how tall they are. If you can't find a counterexample, it DOES NOT mean the conjecture/statement is true. Someone else may come up with one. Lol

Mathematical Conjecture: if $x^2 = 25$, then $x = 5$.

False why?

Counter example: x could also be $x = -5$

After you have read the above...try these exercises on the next page.

Paper and Pencil

- Ask students to provide a counterexample for each conjecture:

  (i) If a number is divisible by 2, then it is divisible by 4.

  (ii) If $x + 4 > 0$, then x is positive.

  (iii) For all positive numbers n, $\frac{1}{n} \leq n$.

  (iv) All linear relationships have one x-intercept.

(NL1.4)

- Ask students to respond to the following:

  (i) For all real numbers x, the expression x^2 is greater than or equal to x. Do you agree? Justify your answer.

  (ii) If $x > 0$, then $\sqrt{x} < x$. Do you agree? Justify your answer.

(NL1.4)

- A mathematician named Goldbach is famous for discovering a conjecture which has not yet been proved to be true or false. Goldbach conjectured that every even number greater than 2 can be written as the sum of two prime numbers. For example, $4 = 2 + 2$, $6 = 3 + 3$, and $8 = 3 + 5$. No one has ever proved this conjecture to be true or found a counterexample to show that it is false, so the conjecture remains just that. As an activity, ask students to make a list of prime numbers at least up to 100, and use them to test the truth of the conjecture for even numbers up to 100.

(NL1.4)