Chapter 1: Set Theory:

- Organizing information into sets and subsets
- Graphically illustrating the relationships between sets and subsets using Venn diagrams
- Solving problems by using sets, subsets and Venn diagrams.

Section 1.1: Types of Sets and Set Notation:

- **Set:** a collection of objects.
  For example, the set of whole numbers is $W = \{0, 1, 2, 3, \ldots\}$

- **Element:** an object in a set. For example, 2 is an element of W

- **Universal Set**: all the elements for the sample.
  For Example, the universal set of digits is
  $D = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$

- **The number of elements in a set**: for example, $n(D) = 10$

- **Subset:** a set whose elements all belong to another set.
  For Example, set Q ,the set of odd digits $Q = \{1, 3, 5, 7, 9\}$ is a subset of set D.
  Using set notation: $Q \subset D$ (Q is a subset of D)

- **Complement:** All the elements of the universal set that do not belong to a subset of it.
  For example, $Q' = \{0, 2, 4, 6, 8\}$ is the complement of Q.
  Notation used is the prime symbol, Q' or not Q

- **Empty Set:** a set with no elements.
  For example the set of odd numbers that are divisible by 2 is the empty set.
  Notation used: { } or Ø

- **Disjoint Sets:** two or more sets having no elements in common.
  For example, the set of even numbers and odd numbers are disjoint.

- **Finite Set:** a set with a countable number of elements.
  For example the set $E = \{2, 4, 6, 8\}$

- **Infinite Set:** a set with an infinite number of elements.
  For example the set of natural numbers, $N = \{1, 2, 3, \ldots\}$

Notation introduced so far:

- Sets are defined using brackets.
  For example to define a universal set with the numbers 1,2 and 3, list its elements: U = {1, 2, 3}

- To define the set A that has the numbers 1 and 2 as elements: $A = \{1, 2\}$

- All elements of A are also elements of U, so A is a subset of U: $A \subset U$

- The set A', the complement of A, can be defined as: $A' = \{3\}$

- To define set ***B***, a subset of U that contains the number 4: $B = \{\ \}$ or $B = \emptyset$
  $B \subset U$

Set Notation

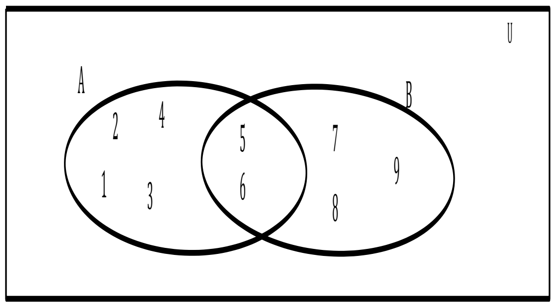
You can represent a set by

- Listing the elements: ex. $A = \{1,2,3,4,5\}$
- Using words: A represents all integers from 1 to 5
- Using set notation: $A = \{x \mid 1 \leq x \leq 5, x \in I\}$

Example: Represent using set notation in two different ways:
Multiples of 2 from 1 to 20:

Venn Diagram can be used to show how sets and subsets are related.

Example 1:



U =

A =

B =

$n(A)$ =

$n(B)$ =

Example 2: Set A = {multiples of 4} Set B = {multiples of 8}
Is $A \subset B$ or is $B \subset A$?

Practice Questions text page 15-18

2. a) Draw a Venn diagram to represent these sets:
- the universal set U = {natural numbers from 1 to 40 inclusive}
- E = {multiples of 8}
- F = {multiples of 4}
- S = {multiples of 17}

b) List the disjoint subsets, if there are any.

c) Is each statement true or false? Explain.

i) $E \subset F$

ii) $F \subset E$

iii) $E \subset E$

iv) F' = {odd numbers from 1 to 40}

v) In this example, the set of natural numbers from 41 to 50 is { }.

6. Consider the following information:
- the universal set U = {natural numbers from 1 to 100 000}
- $X \subset U$
- $n(X) = 12$

Determine $n(X')$, if possible. If it is not possible, explain why.

7. Consider the following information:
- the universal set U = {all natural numbers from 1 to 10 000}
- set X, which is a subset of U
- set Y, which is a subset of U
- $n(X) = 4500$

Determine $n(Y)$, if possible. If it is not possible, explain why.

9. Consider this universal set:

A = {A, B, C, D, E, F, G, H, I, J, K, L, M, N, O, P, Q, R, S, T, U, V, W, X, Y, Z}

a) List the following subsets:
- S = {letters drawn with straight lines only}
- C = {letters drawn with curves only}

b) Is this statement true or false?

$C = S'$

Provide your reasoning.

11. a) Organize the following sets of numbers in a Venn diagram:
- U = {integers from −10 to 10}
- A = {positive integers from 1 to 10 inclusive}
- B = {negative integers from −10 to −1 inclusive}

b) List the disjoint subsets, if there are any.

c) Is each statement true or false? Explain.

i) $A \subset B$

ii) $B \subset A$

iii) $A' = B$

iv) $n(A) = n(B)$

v) For set U, the set of integers from −20 to −15 is { }.

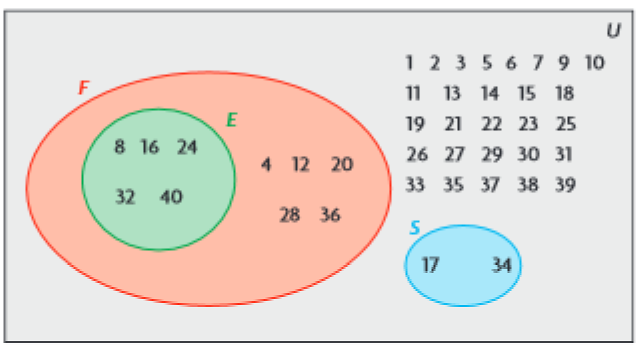
14. Cynthia claims that the $\subset$ sign for sets is similar to the $\leq$ sign for numbers. Explain whether you agree or disagree.

17. a) Organize the following sets in a Venn diagram:
- the universal set R = {real numbers}
- N = {natural numbers}
- W = {whole numbers}
- I = {integers}
- Q = {rational numbers}
- $\overline{Q}$ = {irrational numbers}

b) Identify the complement of each set.
c) Identify any disjoint sets.
d) Are Q' and $\overline{Q}$ equal? Explain.
e) Of which sets is N a subset?
f) Joey drew a Venn diagram to show the sets in part a). In his diagram, the area of set Q was larger than the area of set $\overline{Q}$. Can you conclude that Q has more elements than $\overline{Q}$? Explain.

ANSWERS:

2. a)

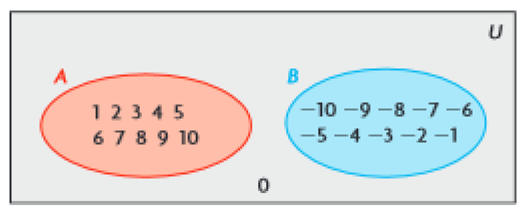


b) E and S, F and S
c) i) True. e.g., Multiples of 8 are also multiples of 4.
ii) False. e.g., Not all multiples of 4 are multiples of 8.
iii) True. e.g., All multiples of 8 are multiples of 8.
iv) False. e.g., F' = {all numbers from 1 to 40 that are not multiples of 4}
v) True. e.g., The universal set includes natural numbers from 1 to 40.

14. e.g., Agree. The number of elements in a subset must be equal to or less than the number of elements in the set.

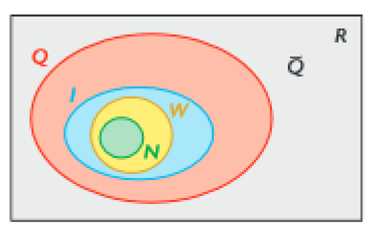
6. 99 988
7. not possible; e.g., There may be some elements that are in both X and Y.
9. a) S = {A, E, F, H, I, K, L, M, N, T, V, W, X, Y, Z}
C = {C, O, S}
b) False. e.g., B is not in S or C.

11. a)



b) A and B
c) i) False. e.g., 1 is not in B.
ii) False. e.g., -1 is not in A.
iii) False. e.g., 0 is in A' but not in B.
iv) True. e.g., $n(A) = 10$, $n(B) = 10$
v) True. e.g., No integer from -20 to -15 is in U.

17. a)



b) e.g., N' is the set of all non-natural numbers. W' is the set of all non-whole numbers. I' is the set of non-integer numbers. Q' is the set of numbers that cannot be described as a ratio of two integers. $\overline{Q}'$ is the set of numbers that can be described as a ratio of two integers.
c) N and $\overline{Q}$, W and $\overline{Q}$, I and $\overline{Q}$, Q and $\overline{Q}$
d) Yes. e.g., Q' is the set of numbers that cannot be described as a ratio of two integers, which is the set of irrational numbers.
e) W, I, Q, R
f) No. e.g., The area of a region in a Venn diagram is not related to the number of elements in the set.

<u>Section 1.2: Exploring Relationships between Sets:</u>

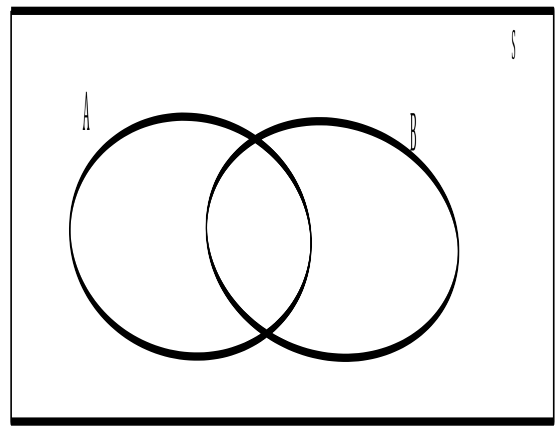
Example 1: Given the universal set S, $S = \{4, 5, 6, 8, 9, 11, 15, 17, 20, 24, 30, 32\}$

(A) place the numbers in the appropriate regions of the Venn Diagram

$A = \{\text{multiples of } 2\}$

$B = \{\text{multiples of } 3\}$

(B) Why is there an overlap?



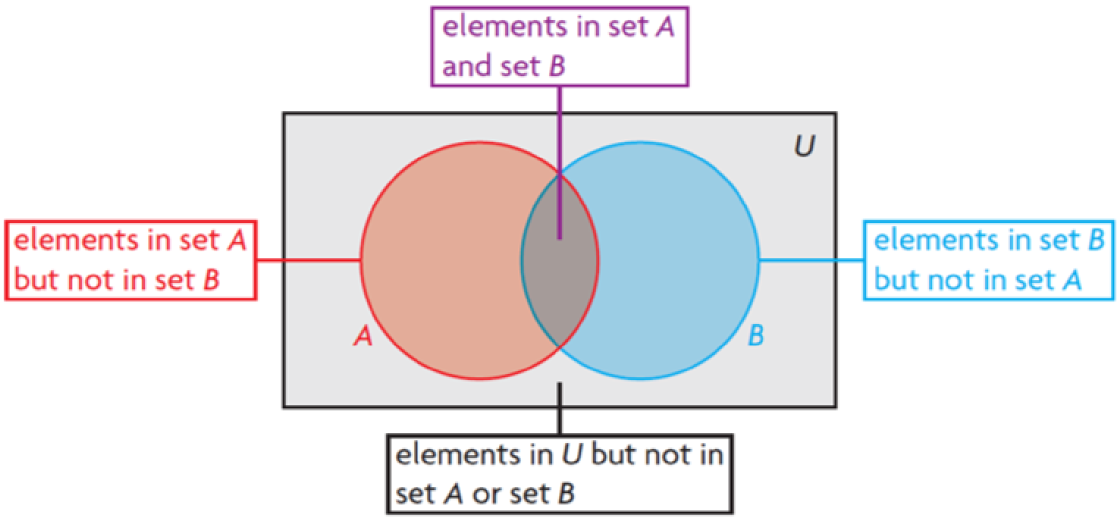
(C) Identify the elements that are in both A and B notation: $A \cap B$

(D) Identify all the elements that are in A or B notation: $A \cup B$

(E) Identify the elements that are not in A and B notation: $(A \cup B)'$

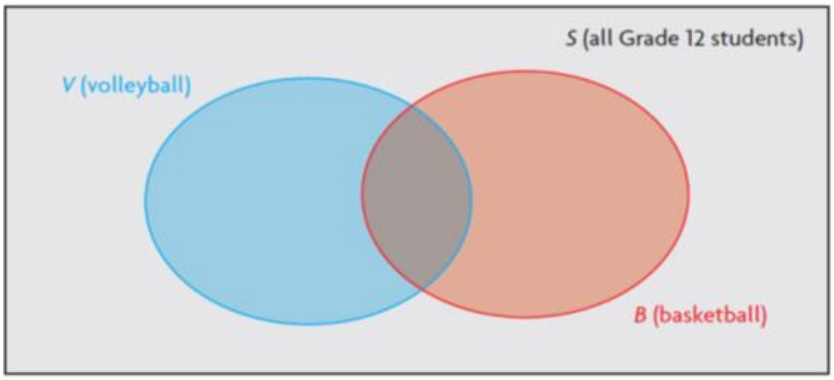
(F) Identify the elements of A that are not part of B notation: $A \backslash B$

- Sets that are not disjoint share common elements
- Each area of a Venn diagram represents something different



Explore:

In a Newfoundland school, there are 65 Grade 12 students. Of these students, 23 play volleyball, and 26 play basketball. There are 31 students who do not play either sport. The following Venn diagram represents the sets of students.



1. Consider the set of students who play volleyball and the set of students who play basketball. Are these two sets disjoint? Explain how you know.

2. Use the Venn Diagram to help determine:

    a. The number of students who play volleyball only.

    b. The number of students who play basketball only.

    c. The number of students who play both volleyball and basketball.

3. Describe how you solved the problem.

Practice Questions P.20 – 21

FURTHER Your Understanding

1. Consider the following sets:

- $U = \{2, 3, 4, 6, 8, 9, 10, 12, 14, 15\}$
- $A = \{3, 6, 9, 12, 15\}$
- $B = \{2, 4, 6, 8, 10, 12, 14\}$

a) Illustrate these sets using a Venn diagram.

b) Determine the number of elements

i) in set A.

ii) in set A but not in set B.

iii) in set B.

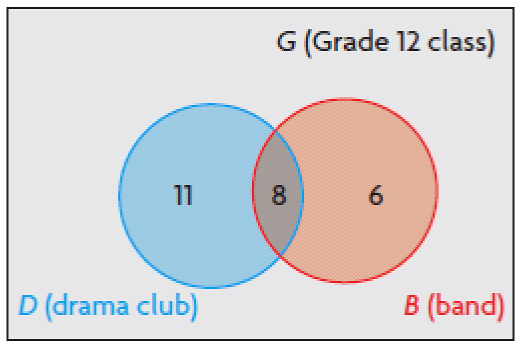
iv) in set B but not in set A.

v) in set A **and** set B.

vi) in set A **or** set B.

vii) in A'.

2. There are 38 students in a Grade 12 class. The number of students in the drama club and the band are illustrated in the Venn diagram. Use the diagram to answer the following questions.



a) How many students are in both the drama club and the band?

b) How many students are in the drama club but not in band?
How many are in the band but not in the drama club?

c) How many students are in the drama club? How many are in the band?

d) How many students are in at least one of the drama club or the band?

e) How many students are in neither the drama club nor the band?

3. Anna surveyed 45 students about their favourite sports. She recorded her results.

Favourite Sports	Number of Students
hockey	20
soccer	14
neither hockey nor soccer	16

a) Determine how many students like hockey and soccer.

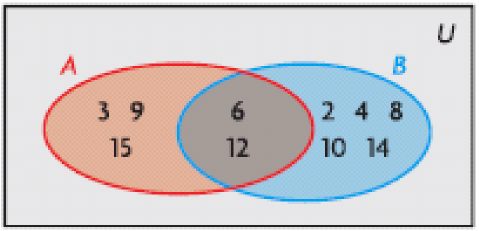
b) Determine how many students like only hockey or only soccer.

c) Draw and label a Venn diagram to show the data.

Answers:

Lesson 1.2, page 20

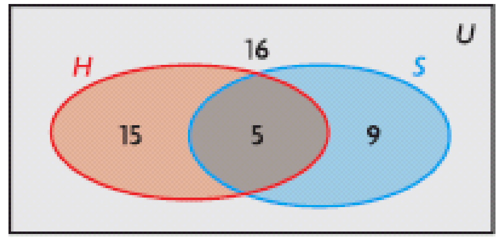
1. **a)**



b) **i)** 5
ii) 3
iii) 7
iv) 5
v) 2
vi) 10
vii) 5

2. **a)** 8
b) 11; 6
c) 19; 14
d) 25
e) 13

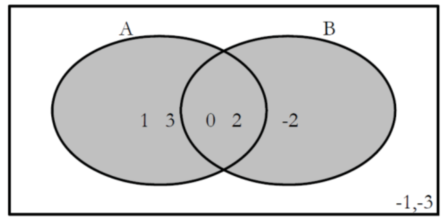
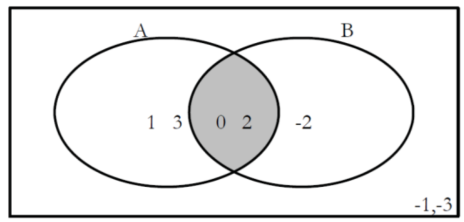
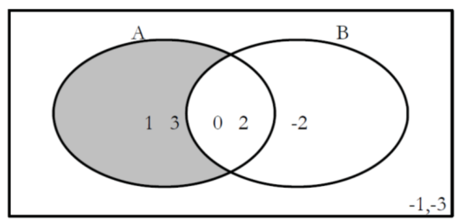
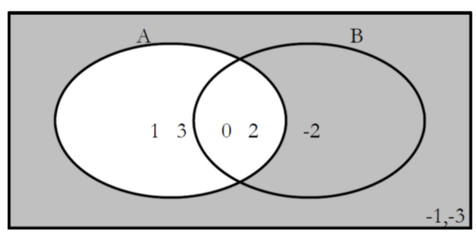
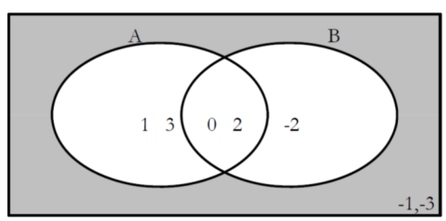
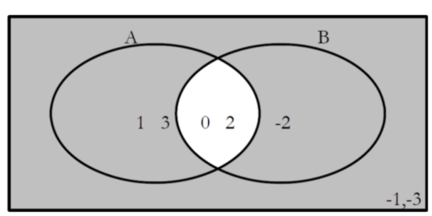
3. **a)** 5
b) 24
c)



Section 1.3: Intersection and Union or Two Sets:

Venn Diagram/Definitions

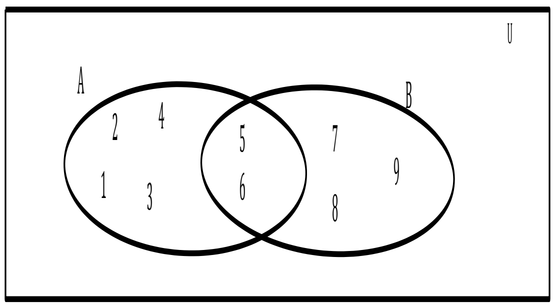
U = set of all integers from -3 to +3 **A** = set of non-negative integers **B** = set of integers divisible by 2

Set Notation	Meaning	Venn Diagram	Answer
$A \cup B$ A union B A **OR** B	Any element that is in either of the sets Any element that is in at least one of the sets		{-2, 0, 1, 2, 3}
$A \cap B$ A intersection B A **AND** B	Only elements that are in both A and B Represented by the overlap region for non-disjoint sets		{0, 2}
A\ B A minus B	Elements found in set A but excluding the ones that are also in set B		{1, 3}
A' A complement Not A	All elements in the universal set outside of A		{-3, -2, -1}
$(A \cup B)'$ Not(A union B)	Elements outside A and B		{-3, -1}
$(A \cap B)'$ Not(A intersect B)	Elements outside of the overlap of A and B		{-3, -2, -1, 1,3}

Venn Diagrams can help develop formulas to determine the number of elements in certain sets.

Example 1:

What formula can be used to determine the $n(A\backslash B)$



There is more than one formula that can be used…
Just as long as it makes sense!

$$n(A\backslash B) = n(A) - n(A \cap B)$$

Or

$$n(A\backslash B) = n(A \cup B) - n(B)$$

Example 2:

Given the following sets: Set A = {2, 3, 6, 8, 9}
Set B = {4, 5, 6, 7, 9}

(A)What elements are in $A \cup B$?

(B) What is the $(A \cup B)$?

(C)The $n(A) = 5$ and the $n(B) = 5$, should the $n(A \cup B) = 10$?

(D) How can you compensate for this over-counting?

The Principle of Inclusion and Exclusion

- If two sets, A and B, contain common elements, to calculate the number of elements in A or B, $n(A \cup B)$, you must subtract the elements in the intersection so that they are not counted twice.

$$n(A \cup B) = n(A) + n(B) - n(A \cap B)$$

- If two sets, A and B, are disjoint, they do not have any intersection.

$$n(A \cap B) = 0 \text{ and}$$

$$n(A \cup B) = n(A) + n(B)$$

- $n(A \cup B)$ can also be determined using $A\backslash B$

$$n(A \cup B) = n(A\backslash B) + n(B\backslash A) + n(A \cap B)$$

Example 3: (Ex.4 page 29)

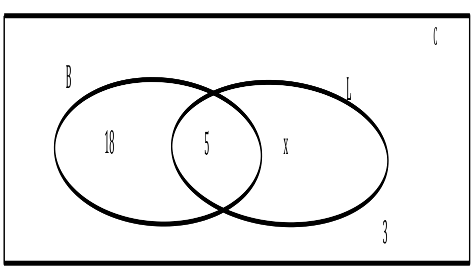
Morgan surveyed the 30 students in her math class about their eating habits.

- 18 of these students eat breakfast
- 5 of the 18 also eat a healthy lunch
- 3 students do not eat breakfast and do not eat a healthy lunch.

How many students eat a healthy lunch?

Tyler solved the problem, as shown below but made an error.

What error did Tyler make? Determine the correct solution.



There are 3 students who don't belong in either region. This means there are 30 – 3 = 27 in B or L

18 + 5 + x = 27

x = 4

therefore,

n(L) = 5 + 4

n(L) = 9

Practice problem:
Page 32-34 #1,3,8,10,15,18

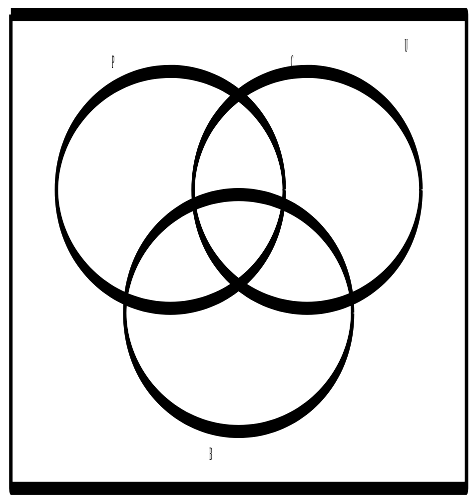
Section 1.4: Applications of Set Theory:

Working with 3 sets in a Venn Diagram:

- Start at center where 3 sets intersect.
- Reminders the numbers in all the regions total the number in the universal set.
- Careful of wording. “only” means just that region

Example 1:

There are 36 students who study science.
14 study Physics
18 study chemistry
24 study biology
5 study physics and chemistry
8 study physics and biology
10 study biology and chemistry
3 study all three subjects.

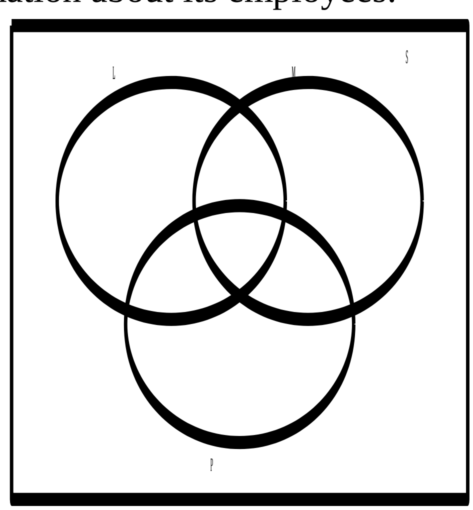


Determine the number of students who

A) Study physics and biology only
B) Study at least two subjects
C) Study biology only

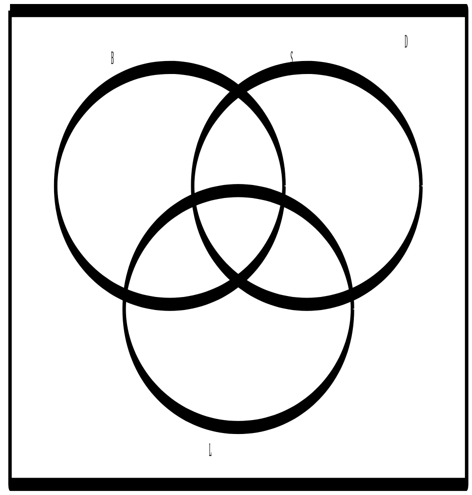
Example 2:

A survey of a machine shop reveals the following information about its employees:
44 can run a lathe
49 can run a milling machine
56 can operate a press punch
27 can run a lathe and milling machine
19 can run a milling machine and operate a press punch
24 can run a lathe and operate a press punch
10 can operate all three machines.
9 cannot operate any of the machines
How many people are employed at the machine shop?



Example 3:

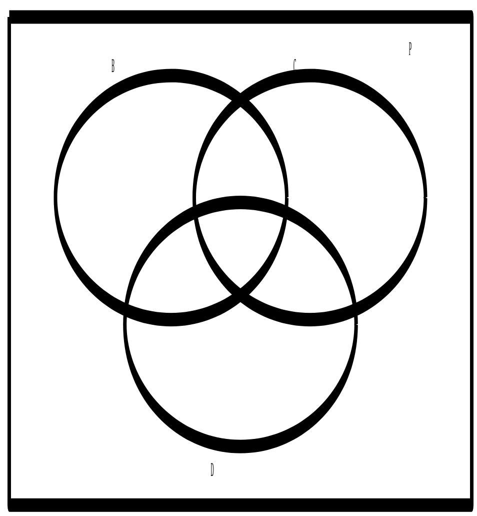
There are 25 dogs at the dog show.
12 dogs are black, 8 dogs have a short tail, 15 dogs have long hair
1 dog is black with a short tail and long hair
3 dogs are black with short tails but do not have long hair
2 dogs have short tails and long hair but are not black.
If all the dogs in the kennel have at least one of the mentioned characteristics, how many dogs are black with long hair but do not have short tails?



Example 4:

28 children have a dog, a cat, or a bird
13 children have a dog, 13 children have a cat, and 13 children have a bird.
4 children have only a dog and a cat
3 children have only a dog and a bird
2 children have only a cat and a bird
No child has two of each type of pet.

A) How many children have a cat, a dog, and a bird?
B) How many children have only 1 pet?



Practice problem:
Page 51-54 #2,4,6,9,13,14