Mathematics

Academic Mathematics 2201

Interim Edition

Newfoundland Labrador

Curriculum Guide
2012

Contents

Acknowledgements

The Department of Education would like to thank the Western and Northern Canadian Protocol (WNCP) for Collaboration in Education, *The Common Curriculum Framework for K-9 Mathematics* - May 2006 and the Common Curriculum Framework for Grades 10-12 - January 2008, reproduced and/or adapted by permission. All rights reserved.

We would also like to thank the provincial Grade 11 Academic Mathematics curriculum committee and the following people for their contribution:

Deanne Lynch, Program Development Specialist - Mathematics Division of Program Development, Department of Education

Joanne Hogan, Program Development Specialist - Mathematics Division of Program Development, Department of Education

John-Kevin Flynn, Test Development Specialist - Mathematics/Science, Division of Evaluation and Research, Department of Education

Mark Barter, Teacher - Holy Heart of Mary High, St.John's

Sheldon Critch, Teacher - Botwood Collegiate, Botwood

Bernard Gushue, Teacher - Pasadena Academy, Pasadena

Gord Melindy, Teacher - Pearson Academy, Wesleyville

Desmond Montague, Teacher - Lake Melville School, North West River

Karen Skinner, Teacher - Corner Brook Regional High School, Corner Brook

Melissa Spurrel, Teacher - Clarenville High School, Clarenville

Linda Stacey, Teacher - O'Donel High School, Mount Pearl

Every effort has been made to acknowledge all sources that contributed to the devleopment of this document. Any omissions or errors will be amended in future printings.

INTRODUCTION

Background

The curriculum guide communicates high expectations for students.

The Mathematics curriculum guides for Newfoundland and Labrador have been derived from *The Common Curriculum Framework for 10-12 Mathematics: Western and Northern Canadian Protocol*, January 2008. These guides incorporate the conceptual framework for Grades 10 to 12 Mathematics and the general outcomes, specific outcomes and achievement indicators established in the common curriculum framework. They also include suggestions for teaching and learning, suggested assessment strategies, and an identification of the associated resource match between the curriculum and authorized, as well as recommended, resource materials.

Beliefs About Students and Mathematics

Mathematical understanding is fostered when students build on their own experiences and prior knowledge.

Students are curious, active learners with individual interests, abilities and needs. They come to classrooms with varying knowledge, life experiences and backgrounds. A key component in developing mathematical literacy is making connections to these backgrounds and experiences.

Students learn by attaching meaning to what they do, and they need to construct their own meaning of mathematics. This meaning is best developed when learners encounter mathematical experiences that proceed from the simple to the complex and from the concrete to the abstract. Through the use of manipulatives and a variety of pedagogical approaches, teachers can address the diverse learning styles, cultural backgrounds and developmental stages of students, and enhance within them the formation of sound, transferable mathematical understandings. Students at all levels benefit from working with a variety of materials, tools and contexts when constructing meaning about new mathematical ideas. Meaningful student discussions provide essential links among concrete, pictorial and symbolic representations of mathematical concepts.

The learning environment should value and respect the diversity of students' experiences and ways of thinking, so that students feel comfortable taking intellectual risks, asking questions and posing conjectures. Students need to explore problem-solving situations in order to develop personal strategies and become mathematically literate. They must come to understand that it is acceptable to solve problems in a variety of ways and that a variety of solutions may be acceptable.

Program Design and Components

Affective Domain

To experience success, students must learn to set achievable goals and assess themselves as they work toward these goals.

A positive attitude is an important aspect of the affective domain and has a profound impact on learning. Environments that create a sense of belonging, encourage risk taking and provide opportunities for success help develop and maintain positive attitudes and self-confidence within students. Students with positive attitudes toward learning mathematics are likely to be motivated and prepared to learn, participate willingly in classroom activities, persist in challenging situations and engage in reflective practices.

Teachers, students and parents need to recognize the relationship between the affective and cognitive domains, and attempt to nurture those aspects of the affective domain that contribute to positive attitudes. To experience success, students must learn to set achievable goals and assess themselves as they work toward these goals.

Striving toward success and becoming autonomous and responsible learners are ongoing, reflective processes that involve revisiting, asssessing and revising personal goals.

Goals For Students

Mathematics education must prepare students to use mathematics confidently to solve problems.

The main goals of mathematics education are to prepare students to:

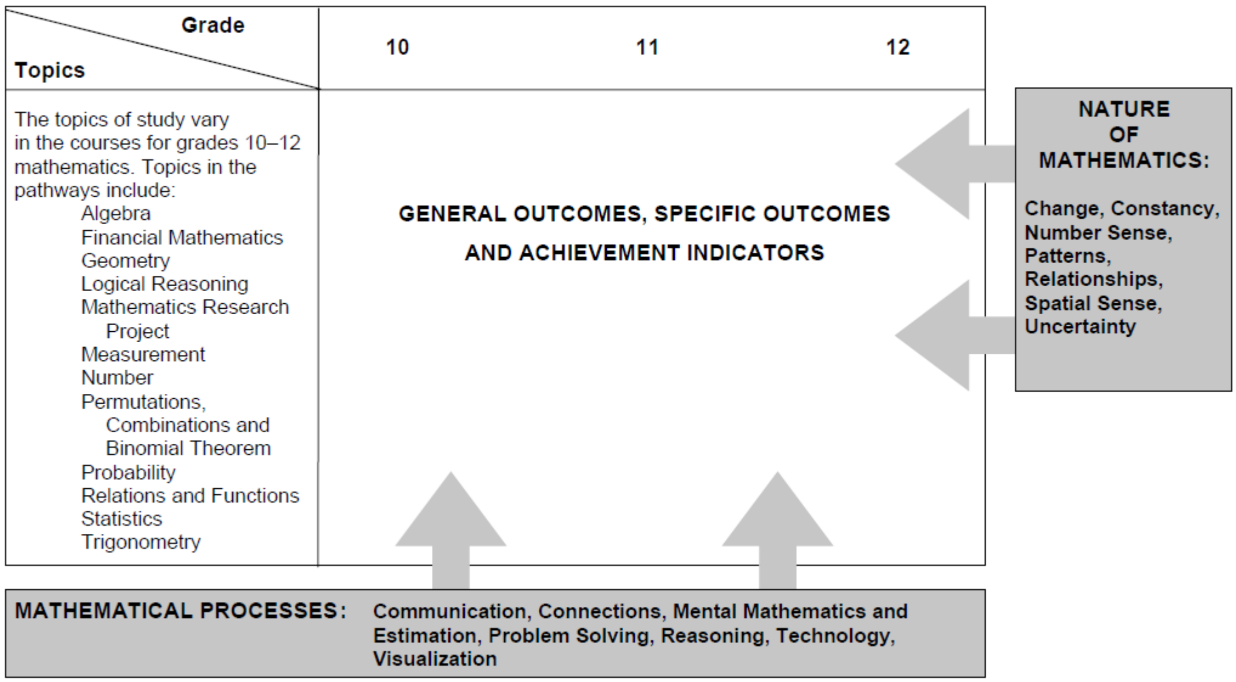
- use mathematics confidently to solve problems
- communicate and reason mathematically
- appreciate and value mathematics
- make connections between mathematics and its applications
- commit themselves to lifelong learning
- become mathematically literate adults, using mathematics to contribute to society.

Students who have met these goals will:

- gain understanding and appreciation of the contributions of mathematics as a science, philosophy and art
- exhibit a positive attitude toward mathematics
- engage and persevere in mathematical tasks and projects
- contribute to mathematical discussions
- take risks in performing mathematical tasks
- exhibit curiosity.

CONCEPTUAL FRAMEWORK FOR 10-12 MATHEMATICS

The chart below provides an overview of how mathematical processes and the nature of mathematics influence learning outcomes.



Mathematical Processes

- *Communication [C]*
- *Connections [CN]*
- *Mental Mathematics and Estimation [ME]*
- *Problem Solving [PS]*
- *Reasoning [R]*
- *Technology [T]*
- *Visualization [V]*

There are critical components that students must encounter in a mathematics program in order to achieve the goals of mathematics education and embrace lifelong learning in mathematics.

Students are expected to:

- communicate in order to learn and express their understanding
- connect mathematical ideas to other concepts in mathematics, to everyday experiences and to other disciplines
- demonstrate fluency with mental mathematics and estimation
- develop and apply new mathematical knowledge through problem solving
- develop mathematical reasoning
- select and use technologies as tools for learning and for solving problems
- develop visualization skills to assist in processing information, making connections and solving problems.

This curriculum guide incorporates these seven interrelated mathematical processes that are intended to permeate teaching and learning.

Communication [C]

Students must be able to communicate mathematical ideas in a variety of ways and contexts.

Students need opportunities to read about, represent, view, write about, listen to and discuss mathematical ideas. These opportunities allow students to create links between their own language and ideas, and the formal language and symbols of mathematics.

Communication is important in clarifying, reinforcing and modifying ideas, attitudes and beliefs about mathematics. Students should be encouraged to use a variety of forms of communication while learning mathematics. Students also need to communicate their learning using mathematical terminology.

Communication helps students make connections among concrete, pictorial, symbolic, oral, written and mental representations of mathematical ideas.

Connections [CN]

Through connections, students begin to view mathematics as useful and relevant.

Contextualization and making connections to the experiences of learners are powerful processes in developing mathematical understanding. When mathematical ideas are connected to each other or to real-world phenomena, students begin to view mathematics as useful, relevant and integrated.

Learning mathematics within contexts and making connections relevant to learners can validate past experiences and increase student willingness to participate and be actively engaged.

The brain is constantly looking for and making connections. "Because the learner is constantly searching for connections on many levels, educators need to *orchestrate the experiences* from which learners extract understanding … Brain research establishes and confirms that multiple complex and concrete experiences are essential for meaningful learning and teaching" (Caine and Caine, 1991, p.5).

Mental Mathematics and Estimation [ME]

Mental mathematics and estimation are fundamental components of number sense.

Mental mathematics is a combination of cognitive strategies that enhance flexible thinking and number sense. It is calculating mentally without the use of external memory aids.

Mental mathematics enables students to determine answers without paper and pencil. It improves computational fluency by developing efficiency, accuracy and flexibility.

"Even more important than performing computational procedures or using calculators is the greater facility that students need—more than ever before—with estimation and mental math" (National Council of Teachers of Mathematics, May 2005).

Students proficient with mental mathematics "... become liberated from calculator dependence, build confidence in doing mathematics, become more flexible thinkers and are more able to use multiple approaches to problem solving" (Rubenstein, 2001, p. 442).

Mental mathematics "... provides the cornerstone for all estimation processes, offering a variety of alternative algorithms and nonstandard techniques for finding answers" (Hope, 1988, p. v).

Estimation is used for determining approximate values or quantities or for determining the reasonableness of calculated values. It often uses benchmarks or referents. Students need to know when to estimate, how to estimate and what strategy to use.

Estimation assists individuals in making mathematical judgements and in developing useful, efficient strategies for dealing with situations in daily life.

Problem Solving [PS]

Learning through problem solving should be the focus of mathematics at all grade levels.

Learning through problem solving should be the focus of mathematics at all grade levels. When students encounter new situations and respond to questions of the type, "How would you know?" or "How could you ...?", the problem-solving approach is being modelled. Students develop their own problem-solving strategies by listening to, discussing and trying different strategies.

A problem-solving activity requires students to determine a way to get from what is known to what is unknown. If students have already been given steps to solve the problem, it is not a problem, but practice. A true problem requires students to use prior learning in new ways and contexts. Problem solving requires and builds depth of conceptual understanding and student engagement.

Problem solving is a powerful teaching tool that fosters multiple, creative and innovative solutions. Creating an environment where students openly seek and engage in a variety of strategies for solving problems empowers students to explore alternatives and develops confident, cognitive mathematical risk takers.

Reasoning [R]

Mathematical reasoning helps students think logically and make sense of mathematics.

Mathematical reasoning helps students think logically and make sense of mathematics. Students need to develop confidence in their abilities to reason and justify their mathematical thinking. High-order questions challenge students to think and develop a sense of wonder about mathematics.

Mathematical experiences in and out of the classroom provide opportunities for students to develop their ability to reason. Students can explore and record results, analyze observations, make and test generalizations from patterns, and reach new conclusions by building upon what is already known or assumed to be true.

Reasoning skills allow students to use a logical process to analyze a problem, reach a conclusion and justify or defend that conclusion.

Technology [T]

Technology contributes to the learning of a wide range of mathematical outcomes and enables students to explore and create patterns, examine relationships, test conjectures and solve problems.

Technology contributes to the learning of a wide range of mathematical outcomes and enables students to explore and create patterns, examine relationships, test conjectures and solve problems.

Technology can be used to:

- explore and demonstrate mathematical relationships and patterns
- organize and display data
- extrapolate and interpolate
- assist with calculation procedures as part of solving problems
- decrease the time spent on computations when other mathematical learning is the focus
- reinforce the learning of basic facts
- develop personal procedures for mathematical operations
- create geometric patterns
- simulate situations
- develop number sense.

Technology contributes to a learning environment in which the growing curiosity of students can lead to rich mathematical discoveries at all grade levels.

Visualization [V]

Visualization is fostered through the use of concrete materials, technology and a variety of visual representations.

Visualization "involves thinking in pictures and images, and the ability to perceive, transform and recreate different aspects of the visual-spatial world" (Armstrong, 1993, p. 10). The use of visualization in the study of mathematics provides students with opportunities to understand mathematical concepts and make connections among them.

Visual images and visual reasoning are important components of number, spatial and measurement sense. Number visualization occurs when students create mental representations of numbers.

Being able to create, interpret and describe a visual representation is part of spatial sense and spatial reasoning. Spatial visualization and reasoning enable students to describe the relationships among and between 3-D objects and 2-D shapes.

Measurement visualization goes beyond the acquisition of specific measurement skills. Measurement sense includes the ability to determine when to measure, when to estimate and which estimation strategies to use (Shaw and Cliatt, 1989).

Nature of Mathematics

- *Change*
- *Constancy*
- *Number Sense*
- *Patterns*
- *Relationships*
- *Spatial Sense*
- *Uncertainty*

Mathematics is one way of trying to understand, interpret and describe our world. There are a number of components that define the nature of mathematics and these are woven throughout this curiculum guide. The components are change, constancy, number sense, patterns, relationships, spatial sense and uncertainty.

Change

Change is an integral part of mathematics and the learning of mathematics.

It is important for students to understand that mathematics is dynamic and not static. As a result, recognizing change is a key component in understanding and developing mathematics.

Within mathematics, students encounter conditions of change and are required to search for explanations of that change. To make predictions, students need to describe and quantify their observations, look for patterns, and describe those quantities that remain fixed and those that change. For example, the sequence 4, 6, 8, 10, 12, … can be described as:

- the number of a specific colour of beads in each row of a beaded design
- skip counting by 2s, starting from 4
- an arithmetic sequence, with first term 4 and a common difference of 2
- a linear function with a discrete domain

(Steen, 1990, p. 184).

Constancy

Constancy is described by the terms stability, conservation, equilibrium, steady state and symmetry.

Different aspects of constancy are described by the terms stability, conservation, equilibrium, steady state and symmetry (AAAS-Benchmarks, 1993, p.270). Many important properties in mathematics and science relate to properties that do not change when outside conditions change. Examples of constancy include the following:

- The ratio of the circumference of a teepee to its diameter is the same regardless of the length of the teepee poles.
- The sum of the interior angles of any triangle is 180°.
- The theoretical probability of flipping a coin and getting heads is 0.5.

Some problems in mathematics require students to focus on properties that remain constant. The recognition of constancy enables students to solve problems involving constant rates of change, lines with constant slope, direct variation situations or the angle sums of polygons.

Number Sense

An intuition about number is the most important foundation of a numerate child.

Number sense, which can be thought of as intuition about numbers, is the most important foundation of numeracy (British Columbia Ministry of Education, 2000, p.146).

A true sense of number goes well beyond the skills of simply counting, memorizing facts and the situational rote use of algorithms. Mastery of number facts is expected to be attained by students as they develop their number sense. This mastery allows for facility with more complex computations but should not be attained at the expense of an understanding of number.

Number sense develops when students connect numbers to their own real-life experiences and when students use benchmarks and referents. This results in students who are computationally fluent and flexible with numbers and who have intuition about numbers. The evolving number sense typically comes as a by product of learning rather than through direct instruction. It can be developed by providing rich mathematical tasks that allow students to make connections to their own expereinces and their previous learning.

Patterns

Mathematics is about recognizing, describing and working with numerical and non-numerical patterns.

Mathematics is about recognizing, describing and working with numerical and non-numerical patterns. Patterns exist in all strands of mathematics.

Working with patterns enables students to make connections within and beyond mathematics. These skills contribute to students' interaction with, and understanding of, their environment.

Patterns may be represented in concrete, visual or symbolic form. Students should develop fluency in moving from one representation to another.

Students must learn to recognize, extend, create and use mathematical patterns. Patterns allow students to make predictions and justify their reasoning when solving routine and non-routine problems.

Learning to work with patterns in the early grades helps students develop algebraic thinking, which is foundational for working with more abstract mathematics.

Relationships

Mathematics is used to describe and explain relationships.

Mathematics is one way to describe interconnectedness in a holistic worldview. Mathematics is used to describe and explain relationships. As part of the study of mathematics, students look for relationships among numbers, sets, shapes, objects and concepts. The search for possible relationships involves collecting and analyzing data and describing relationships visually, symbolically, orally or in written form.

Spatial Sense

Spatial sense offers a way to interpret and reflect on the physical environment.

Spatial sense involves visualization, mental imagery and spatial reasoning. These skills are central to the understanding of mathematics.

Spatial sense is developed through a variety of experiences and interactions within the environment. The development of spatial sense enables students to solve problems involving 3-D objects and 2-D shapes and to interpret and reflect on the physical environment and its 3-D or 2-D representations.

Some problems involve attaching numerals and appropriate units (measurement) to dimensions of shapes and objects. Spatial sense allows students to make predictions about the results of changing these dimensions; e.g., doubling the length of the side of a square increases the area by a factor of four. Ultimately, spatial sense enables students to communicate about shapes and objects and to create their own representations.

Uncertainty

Uncertainty is an inherent part of making predictions.

In mathematics, interpretations of data and the predictions made from data may lack certainty.

Events and experiments generate statistical data that can be used to make predictions. It is important to recognize that these predictions (interpolations and extrapolations) are based upon patterns that have a degree of uncertainty.

The quality of the interpretation is directly related to the quality of the data. An awareness of uncertainty allows students to assess the reliability of data and data interpretation.

Chance addresses the predictability of the occurrence of an outcome. As students develop their understanding of probability, the language of mathematics becomes more specific and describes the degree of uncertainty more accurately.

Essential Graduation Learnings

Essential graduation learnings are statements describing the knowledge, skills and attitudes expected of all students who graduate from high school. Essential graduation learnings are cross-curricular in nature and comprise different areas of learning: *aesthetic expression, citizenship, communication, personal development, problem solving, technological competence* and *spiritual and moral development*.

Aesthetic Expression

Graduates will be able to respond with critical awareness to various forms of the arts and be able to express themselves through the arts.

Citizenship

Graduates will be able to assess social, cultural, economic and environmental interdependence in a local and global context.

Communication

Graduates will be able to use the listening, viewing, speaking, reading and writing modes of language(s) and mathematical and scientific concepts and symbols to think, learn and communicate effectively.

Personal Development

Graduates will be able to continue to learn and to pursue an active, healthy lifestyle.

Problem Solving

Graduates will be able to use the strategies and processes needed to solve a wide variety of problems, including those requiring language and mathematical and scientific concepts.

Technological Competence

Graduates will be able to use a variety of technologies, demonstrate an understanding of technological applications, and apply appropriate technologies for solving problems.

Spiritual and Moral Development

Graduates will be able to demonstrate an understanding and appreciation for the place of belief systems in shaping the development of moral values and ethical conduct.

See *Foundations for the Atlantic Canada Mathematics Curriculum*, pages 4-6.

The mathematics curriculum is designed to make a significant contribution towards students' meeting each of the essential graduation learnings (EGLs), with the communication, problem-solving and technological competence EGLs relating particularly well to the mathematical processes.

Outcomes and Achievement Indicators

The curriculum is stated in terms of general outcomes, specific outcomes and achievement indicators.

General Outcomes

General outcomes are overarching statements about what students are expected to learn in each course.

Specific Outcomes

Specific outcomes are statements that identify the specific skills, understanding and knowledge that students are required to attain by the end of a given course.

In the specific outcomes, the word *including* indicates that any ensuing items must be addressed to fully meet the learning outcome. The phrase *such as* indicates that the ensuing items are provided for illustrative purposes or clarification, and are not requirements that must be addressed to fully meet the learning outcome.

Achievement Indicators

Achievement indicators are samples of how students may demonstrate their achievement of the goals of a specific outcome. The range of samples provided is meant to reflect the scope of the specific outcome.

Specific curriculum outcomes represent the means by which students work toward accomplishing the general curriculum outcomes and ultimately, the essential graduation learnings.

Program Organization

<table>
<tr><th>Program Level</th><th>Course 1</th><th>Course 2</th><th>Course 3</th><th>Course 4</th></tr>
<tr><td>Advanced</td><td rowspan="2">Mathematics 1201</td><td>Mathematics 2200</td><td>Mathematics 3200</td><td>Mathematics 3208</td></tr>
<tr><td>Academic</td><td>Mathematics 2201</td><td>Mathematics 3201</td><td></td></tr>
<tr><td>Applied</td><td>Mathematics 1202</td><td>Mathematics 2202</td><td>Mathematics 3202</td><td></td></tr>
</table>

The applied program is designed to provide students with the mathematical understandings and critical-thinking skills identified for entry into the majority of trades and for direct entry into the workforce.

The academic and advanced programs are designed to provide students with the mathematical understandings and critical-thinking skills identified for entry into post-secondary programs. Students who complete the advanced program will be better prepared for programs that require the study of calculus.

The programs aim to prepare students to make connections between mathematics and its applications and to become numerate adults, using mathematics to contribute to society.

Summary

The conceptual framework for Grades 10-12 Mathematics (p.3) describes the nature of mathematics, mathematical processes and the mathematical concepts to be addressed. The components are not meant to stand alone. Activities that take place in the mathematics classroom should result from a problem-solving approach, be based on mathematical processes and lead students to an understanding of the nature of mathematics through specific knowledge, skills and attitudes among and between topics.

ASSESSMENT AND EVALUATION

Purposes of Assessment

What learning is assessed and evaluated, how it is assessed and evaluated, and how results are communicated send clear messages to students and others about what is really valued.

Assessment techniques are used to gather information for evaluation. Information gathered through assessment helps teachers determine students' strengths and needs in their achievement of mathematics and guides future instructional approaches.

Teachers are encouraged to be flexible in assessing the learning success of all students and to seek diverse ways in which students might demonstrate what they know and are able to do.

Evaluation involves the weighing of the assessment information against a standard in order to make an evaluation or judgment about student achievement.

Assessment has three interrelated purposes:

- assessment *for* learning to guide and inform instruction;
- assessment *as* learning to involve students in self-assessment and setting goals for their own learning; and
- assessment *of* learning to make judgements about student performance in relation to curriculum outcomes.

Assessment *for* Learning

Assessment *for* learning involves frequent, interactive assessments designed to make student understanding visible. This enables teachers to identify learning needs and adjust teaching accordingly. It is an ongoing process of teaching and learning.

Assessment *for* learning:

- requires the collection of data from a range of assessments as investigative tools to find out as mush as possible about what students know
- provides descriptive, specific and instructive feedback to students and parents regarding the next stage of learning
- actively engages students in their own learning as they assess themselves and understand how to improve performance.

Assessment *as* Learning

Assessment *as* learning actively involves students' reflection on their learning and monitoring of their own progress. It focuses on the role of the student as the critical connector between assessment and learning, thereby developing and supporting metacognition in students.

Assessment *as* learning:

- supports students in critically analysing their learning related to learning outcomes
- prompts students to consider how they can continue to improve their learning
- enables students to use information gathered to make adaptations to their learning processes and to develop new understandings.

Assessment *of* Learning

Assessment *of* learning involves strategies to confirm what students know, demonstrate whether or not they have met curriculum outcomes, or to certify proficiency and make decisions about students' future learning needs. Assessment *of* learning occurs at the end of a learning experience that contributes directly to reported results.

Traditionally, teachers relied on this type of assessment to make judgments about student performance by measuring learning after the fact and then reporting it to others. Used in conjunction with the other assessment processes previously outlined, however, assessment *of* learning is strengthened.

Assessment *of* learning:

- provides opportunities to report evidence to date of student achievement in relation to learning outcomes, to parents/guardians and other stakeholders
- confirms what students know and can do
- occurs at the end of a learning experience using a variety of tools.

Because the consequences of assessment *of* learning are often far-reaching, teachers have the responsibility of reporting student learning accurately and fairly, based on evidence obtained from a variety of contexts and applications.

Assessment Strategies

Assessment techniques should match the style of learning and instruction employed. Several options are suggested in this curriculum guide from which teachers may choose, depending on the curriculum outcomes, the class and school/district policies.

Observation (formal or informal)

This technique provides a way of gathering information fairly quickly while a lesson is in progress. When used formally, the student(s) would be aware of the observation and the criteria being assessed. Informally, it could be a frequent, but brief, check on a given criterion. Observation may offer information about the participation level of a student for a given task, use of a concrete model or application of a given process. The results may be recorded in the form of checklists, rating scales or brief written notes. It is important to plan in order that specific criteria are identified, suitable recording forms are ready, and all students are observed within a reasonable period of time.

Performance

This curriculum encourages learning through active participation. Many of the curriculum outcomes promote skills and their applications. In order for students to appreciate the importance of skill development, it is important that assessment provide feedback on the various skills. These may be the correct manner in which to use a manipulative, the ability to interpret and follow instructions, or to research, organize and present information. Assessing performance is most often achieved through observing the process.

Paper and Pencil

These techniques can be formative or summative. Whether as part of learning, or a final statement, students should know the expectations for the exercise and how it will be assessed. Written assignments and tests can be used to assess knowledge, understanding and application of concepts. They are less successful at assessing processes and attitudes. The purpose of the assessment should determine what form of paper and pencil exercise is used.

Journal

Journals provide an opportunity for students to express thoughts and ideas in a reflective way. By recording feelings, perceptions of success, and responses to new concepts, a student may be helped to identify his or her most effective learning style. Knowing how to learn in an effective way is powerful information. Journal entries also give indicators of developing attitudes to mathematical concepts, processes and skills, and how these may be applied in the context of society. Self-assessment, through a journal, permits a student to consider strengths and weaknesses, attitudes, interests and new ideas. Developing patterns may help in career decisions and choices of further study.

Interview

This curriculum promotes understanding and applying mathematics concepts. Interviewing a student allows the teacher to confirm that learning has taken place beyond simple factual recall. Discussion allows a student to display an ability to use information and clarify understanding. Interviews may be a brief discussion between teacher and student or they may be more extensive. Such conferences allow students to be proactive in displaying understanding. It is helpful for students to know which criteria will be used to assess formal interviews. This assessment technique provides an opportunity to students whose verbal presentation skills are stronger than their written skills.

Presentation

The curriculum includes outcomes that require students to analyze and interpret information, to be able to work in teams, and to communicate information. These activities are best displayed and assessed through presentations. These can be given orally, in written/pictorial form, by project summary, or by using electronic systems such as video or computer software. Whatever the level of complexity, or format used, it is important to consider the curriculum outcomes as a guide to assessing the presentation. The outcomes indicate the process, concepts and context for which a presentation is made.

Portfolio

Portfolios offer another option for assessing student progress in meeting curriculum outcomes over a more extended period of time. This form of assessment allows the student to be central to the process. There are decisions about the portfolio, and its contents, which can be made by the student. What is placed in the portfolio, the criteria for selection, how the portfolio is used, how and where it is stored, and how it is evaluated are some of the questions to consider when planning to collect and display student work in this way. The portfolio should provide a long-term record of growth in learning and skills. This record of growth is important for individual reflection and self-assessment, but it is also important to share with others. For all students, it is exciting to review a portfolio and see the record of development over time.

INSTRUCTIONAL FOCUS

Planning for Instruction

Consider the following when planning for instruction:

- Integration of the mathematical processes within each topic is expected.
- By decreasing emphasis on rote calculation, drill and practice, and the size of numbers used in paper and pencil calculations, more time is available for concept development.
- Problem solving, reasoning and connections are vital to increasing mathematical fluency and must be integrated throughout the program.
- There should be a balance among mental mathematics and estimation, paper and pencil exercises, and the use of technology, including calculators and computers. Concepts should be introduced using manipulatives and be developed concretely, pictorially and symbolically.
- Students bring a diversity of learning styles and cultural backgrounds to the classroom. They will be at varying developmental stages.

Teaching Sequence

The curriculum guide for Academic Mathematics 2201 is organized by units. This is only a suggested teaching order for the course. There are a number of combinations of sequences that would be appropriate.

Each two page spread lists the topic, general outcome, and specific outcome.

Instruction Time Per Unit

The suggested number of hours of instruction per unit is listed in the guide at the beginning of each unit. The number of suggested hours includes time for completing assessment activities, reviewing and evaluating. The timelines at the beginning of each unit are provided to assist in planning. The use of these timelines is not mandatory. However, it is mandatory that all outcomes are taught during the school year, so a long term plan is advised. Teaching of the outcomes is ongoing, and may be revisited as necessary.

Resources

The authorized resource for Newfoundland and Labrador students and teachers is *Principles of Mathematics 11* (Nelson). Column four of the curriculum guide references *Principles of Mathematics 11* for this reason. Teachers may use any other resource, or combination of resources, to meet the required specific outcomes.

GENERAL AND SPECIFIC OUTCOMES

GENERAL AND SPECIFIC OUTCOMES WITH ACHIEVEMENT INDICATORS (pages 19-250)

This section presents general and specific outcomes with corresponding achievement indicators and is organized by unit. The list of indicators contained in this section is not intended to be exhaustive but rather to provide teachers with examples of evidence of understanding that may be used to determine whether or not students have achieved a given specific outcome. Teachers may use any number of these indicators or choose to use other indicators as evidence that the desired learning has been achieved. Achievement indicators should also help teachers form a clear picture of the intent and scope of each specific outcome.

Academic Mathematics 2201 is organized into nine units: *Inductive and Deductive Reasoning, Mathematics Inquiry, Properties of Angles and Triangles, Acute Triangle Trigonometry, Radicals, Statistical Reasoning, Quadratic Functions, Quadratic Equations,* and *Proportional Reasoning*.

Inductive and Deductive Reasoning

Suggested Time: 10 Hours

Unit Overview

Focus and Context

Inductive reasoning is a fundamental part of mathematics. A scientist, for example, will use inductive reasoning to formulate a hypothesis based on a pattern in experimental results. In this unit, students will use inductive reasoning to make a generalization or conjecture based on a limited number of observations or examples. They will develop the concept that conjectures are valid unless a counterexample can be found. In this case, either the conjecture is revised or a new conjecture is developed.

Students will then prove their conjectures through the use of deductive reasoning. Deductive reasoning starts with a known fact and creates a specific conclusion from that generalization.

Students will also examine various problems, information, puzzles and games to develop their resoning skills.They will determine, explain and verify a reasoning strategy to solve a puzzle or win a game.

Outcomes Framework

GCO

Develop number sense and logical reasoning.

SCO NL1

Analyze and prove conjectures, using inductive and deductive reasoning, to solve problems.

SCO NL2

Analyze puzzles and games that involve spatial reasoning, using problem-solving strategies.

Process Standards Key

[**C**] Communication
[**CN**] Connections
[**ME**] Mental Mathematics and Estimation
[**PS**] Problem Solving
[**R**] Reasoning
[**T**] Technology
[**V**] Visualization

SCO Continuum

Mathematics 1201	Mathematics 2201	Mathematics 3201
	Number and Logic	Logical Reasoning
	NL1 Analyze and prove conjectures, using inductive and deductive reasoning, to solve problems. [C, CN, PS, R] NL2 Analyze puzzles and games that involve spatial reasoning, using problem-solving strategies. [CN, PS, R, V]	LR1 Analyze puzzles and games that involve numerical and logical reasoning, using problem-solving strategies. [CN, ME, PS, R]

Number and Logic

Outcomes

Students will be expected to

NL1 Analyze and prove conjectures, using inductive and deductive reasoning, to solve problems.

[C, CN, PS, R]

Achievement Indicator:

NL1.1 *Make conjectures by observing patterns and identifying properties, and justify the reasoning.*

Elaborations—Strategies for Learning and Teaching

Inductive reasoning is a form of reasoning in which a conclusion is reached based on a pattern present in numerous observations. The premises makes the conclusion likely, but does not guarantee it to be true. Deductive reasoning is the process of coming up with a conclusion based on facts that have already been shown to be true. The facts that can be used to prove your conclusion deductively may come from accepted definitions, properties, laws or rules. The truth of the premises guarantees the truth of the conclusion.

In this unit, students will examine situations, information, problems, puzzles and games to develop their reasoning skills. They will form conjectures through the use of inductive reasoning and prove their conjectures through the use of deductive reasoning.

A conjecture is a testable expression that is based on available evidence but is not yet proven. Students could be introduced to this unit through the analysis of real-life situations. Given the visual below, for example, ask students to analyze the picture and develop an explanation for the possible events that have occurred. As students explore this example, they will realize they already have had some experience making conjectures. They often draw conclusions by observing patterns and identifying properties in specific examples.

http:/journalismblackville92.blogspot.com

Students should be given an opportunity to make conjectures using a variety of strategies. Encourage them to draw pictures, construct tables, use measuring instruments such as rulers and protractors and/or use geometry software such as FX Draw or Geometer Sketchpad. Dynamic geometry programs, such as these can be used to construct accurate diagrams of geometrical objects. Once students have been exposed to these programs, more complicated conjectures can be investigated which could, in turn, lead to valuable classroom discussions.

General Outcome: Develop number sense and logical reasoning.

Suggested Assessment Strategies

Paper and Pencil

- Ask students to work in groups of three. The first student writes a number or draws a shape. The next student writes a number or draws a shape, beginning a pattern. Continue until each student in the group has contributed to the pattern. Then ask the first student to find the 5^{th} term. Continue this activity until each student has gone first.

(NL1.1)

- Ask students to draw any quadrilateral. Find the midpoint of each side and connect them. Ask them what type of quadrilateral results. They should compare their findings with other classmates and make a conjecture based on their observations.

(NL1.1)

- Ask students to use inductive reasoning to make a conjecture from each of the following situations:

  (i) Mr. Baker wore a black tie the last four days it has been snowing. Today it is snowing. Make a conjecture about the color of his tie.

  (ii) Given the Fibonacci sequence 1, 1, 2, 3, 5, 8, ..., make a conjecture about the 8^{th} term.

(NL1.1)

Presentation

- Ask students to find a pattern in a magazine or newspaper and describe it in words. Ask them to write a conjecture about the pattern and present their example to the class.

(NL1.1)

Journal

- Ask students to describe situations in their lives where they make conjectures.

(NL1.1)

Resources/Notes

Authorized Resource

Principles of Mathematics 11

1.1: Making Conjectures: Inductive Reasoning

Student Book (SB): pp.6-15
Teacher Resource (TR): pp.7-13

Number and Logic

Outcomes

Students will be expected to

NL1 Continued ...

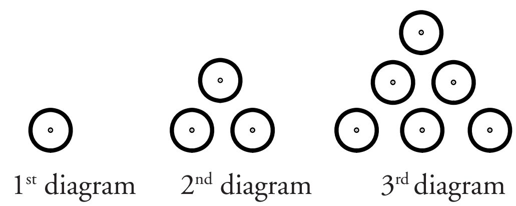
Achievement Indicator:

NL1.1 *Continued*

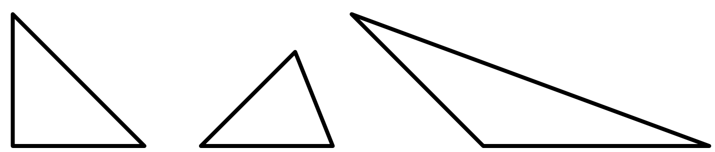
Elaborations—Strategies for Learning and Teaching

Provide students with examples where they can apply the strategies (diagram, table, measuring instrument, etc.) to make a conjecture. Consider the following examples:

(1) What would be the 6th diagram in the following sequence of diagrams?



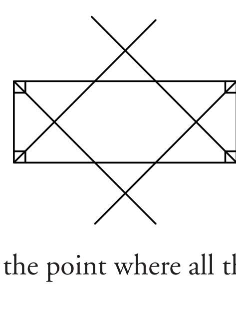
(2) Make a conjecture about the sum of the angles in a triangle.



(3) Make a conjecture about the product of two consecutive integers.

Provide students with specific criteria and encourage them to generate their own conjectures. For example, give students an opportunity to choose a class of quadrilaterals and ask them to identify any properties of the angle bisectors or perpendicular bisectors of the sides of those figures. A sample of answers may include the following:

- The angle bisectors of a rectangle make a square.



- In a kite, the point where all the angle bisectors meet is the center.
- In a trapezoid, the two base angle bisectors form an isosceles triangle at the point of intersection.

Students should be encouraged to share their answers with other classmates, to discuss what they observe, and to explain how they came up with their conjectures.

General Outcome: Develop number sense and logical reasoning.

Suggested Assessment Strategies

Paper and Pencil

- Ask students to use inductive reasoning to make a conjecture from each of the following:

  (i) Using the data shown below (www.tides.gc.ca), decide on the best time to go mussel picking on Saturday.

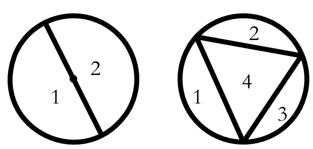
La Scie (Station #1105)
7 days Tidal Prediction
Reference : Chart Datum

Times and Heights for High and Low Tides

(Wednesday)			(Thursday)			(Friday)		
Time	Height		Time	Height		Time	Height	
NDT	(m)	(ft)	NDT	(m)	(ft)	NDT	(m)	(ft)
00:59	**0.1**	0.3	01:36	**0.0**	0.0	02:16	**0.0**	0.0
07:24	**1.3**	4.3	08:05	**1.4**	4.6	08:47	**1.4**	4.6
13:39	**0.0**	0.0	14:21	**-0.0**	0.0	15:02	**0.0**	0.0
19:47	**1.2**	3.9	20:32	**1.2**	3.9	21:21	**1.1**	3.6

(NL1.1)

  (ii) Points are placed on the circumference of a circle and joined. Make a prediction about the number of regions formed when 6 points are used.



(NL1.1)

  (iii) Complete the conjecture started below that holds for all the equations.

  3 + 7 = 10
  11 + 5 = 4
  9 + 13 = 22
  7 + 11 = 18

  Conjecture: The sum of two odd numbers is always an...

(NL1.1)

  (iv) Write a conjecture about combining two numbers using a mathematical operation. Show a number of examples supporting your conjecture.

(NL1.1)

Resources/Notes

Authorized Resource

Principles of Mathematics 11

1.1: Making Conjectures: Inductive Reasoning

SB: pp.6-15
TR: pp.7-13

Number and Logic

Outcomes

Students will be expected to

NL1 Continued ...

Achievement Indicators:

NL1.1 *Continued*

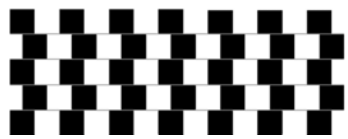
NL1.2 *Explain why inductive reasoning may lead to a false conjecture.*

NL1.3 *Determine if a given argument is valid, and justify the reasoning.*

Elaborations—Strategies for Learning and Teaching

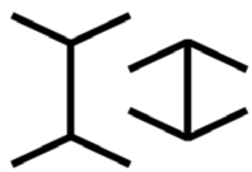
Students should be presented with situations where they are expected to make a conjecture and then verify or discredit it. They will be expected to gather additional evidence, such as taking measurements, performing calculations, or extending patterns, to determine if a given argument is valid. Consider the following examples:

- Make a conjecture about the following lines.



  Students may initially believe that the lines are slanted. Using a ruler to investigate their conjecture, they will discover that the lines are in fact straight.

- Make a conjecture about the length of the line segments.



  In this example, students may make a conjecture that the line segment on the left is longer than the one on the right. If they use a ruler to test their conjecture they will discover that both lines are actually the same length. Students should then revise their conjecture.

Students should develop an understanding that sometimes inductive reasoning may lead to a false conjecture. They can explore situations where the collection of additional data, for example, refutes the conjecture. Consider an example such as the following:

An auto assembly plant can produce a maximum of 400 cars per day. A production line technician records the number of cars coming off the assembly line each hour. The following table gives a record of the technician's observations:

Hours	1	2	3
Cars	10	20	40

Robert conjectures that the number of cars coming off the assembly line doubles each hour. Is this conjecture valid? Explain.

Students may initially believe that this conjecture is valid, but upon further exploration they will reach a number that would exceed the production capacity of the plant.

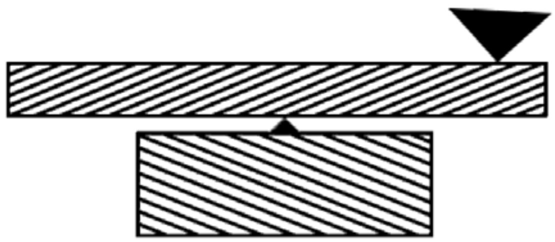
General Outcome: Develop number sense and logical reasoning.

Suggested Assessment Strategies

Paper and Pencil

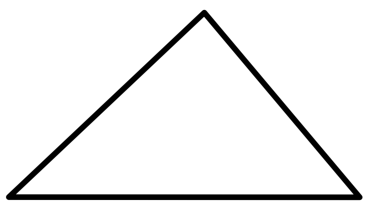
- Ask students to answer the following:

  (i) Susie conjectures that this balance is not level. Do you agree or disagree? Justify.



(NL1.1, NL1.3)

  (ii) Grace conjectures that the triangle below is a right triangle. Do you agree or disagree? Explain.



(NL1.1, NL1.3)

- Ask student to provide an example where a limited number of observations might lead to a false conjecture.

(NL1.2)

Presentation

- Show students a video clip of *Young Sherlock Holmes* to illustrate inductive reasoning. Ask students to note as many examples of inductive reasoning that they hear and identify which ones may lead to a false conjecture.

(NL1.2, NL1.3)

Interview

- Mike had breakfast at Cora's restaurant three Saturdays in a row. He saw Janet there each time. Mike told his friends that Janet always eats breakfast at Cora's on Saturdays. Ask students if this argument is valid? Explain.

(NL1.2, NL1.3)

Resources/Notes

Authorized Resource

Principles of Mathematics 11

1.2: Exploring the Validity of Conjectures

SB: pp.16-17
TR: pp.14-16

Web Link

The following site provides a clip from *Young Sherlock Holmes*:

http://movieclips.com/WijX-young-sherlock-holmes-movie-watson-meets-holmes

Number and Logic

Outcomes

Students will be expected to

NL1 Continued ...

Achievement Indicator:

NL1.4 *Provide and explain a counterexample to disprove a given conjecture.*

Elaborations—Strategies for Learning and Teaching

When students make a conjecture, they will attempt to either prove their conjecture by constructing a valid argument or they may disprove their conjecture by finding a counterexample. One counterexample is sufficient to disprove a conjecture.

To introduce this topic, students could be asked to come up with counterexamples for statements such as the following:

(i) Any piece of furniture having four legs is a table.

(ii) If something has a knob on it, it's a faucet.

(iii) If the grass is wet, it is raining.

(iv) All students like sports.

Students will continue to use strategies, such as drawing pictures or using measuring instruments, when looking for a counterexample. After a counterexample is found, students should revise their conjecture to include the new evidence.

When testing conjectures involving numbers, students may be unable to find a counterexample because they often use the same type of number. For example, students may only consider natural numbers. They should be encouraged to try various types of numbers, such as whole numbers and fractions, positive numbers, negative numbers and zero.

It is important for students to realize that if they cannot find a counterexample this does not prove the conjecture. A mathematician named Goldbach is famous for discovering a conjecture which has not yet been proved to be true or false. Goldbach conjectured that every even number greater than 2 can be written as the sum of two prime numbers. For example, $4 = 2 + 2$, $6 = 3 + 3$, and $8 = 3 + 5$. No one has ever proved this conjecture to be true or found a counterexample to show that it is false, so the conjecture remains just that. As an activity, students can make a list of prime numbers at least up to 100, and use them to test the truth of the conjecture for even numbers up to 100.

General Outcome: Develop number sense and logical reasoning.

Suggested Assessment Strategies

Paper and Pencil

- Ask students to provide a counterexample for each of the following conjectures:

  (i) If a number is divisible by 2, then it is divisible by 4.

  (ii) If $x + 4 > 0$, then x is positive.

  (iii) For all positive numbers n, $\frac{1}{n} \leq n$.

(NL1.4)

- Ask students to respond to the following:

  (i) For all real numbers x, the expression x^2 is greater than or equal to x. Do you agree? Justify your answer.

  (ii) If $x > 0$, then $\sqrt{x} < x$. Do you agree? Justify your answer.

(NL1.4)

Performance

- Ask students to participate in the activity, *Find Your Partner*. Half of the students should be given a card with a conjecture on it and the other half should be given a card with a counterexample on it. Students need to move around the classroom to match the conjecture with the correct counterexample. They should then present their findings to the class.

(NL1.4)

Portfolio

- Ask students to research either a scientific or mathematical theory that was disproven using counterexamples. Ask them to explain the theory and how it was disproven. (This could be part of a research paper for this course).

(NL1.4)

Resources/Notes

Authorized Resource

Principles of Mathematics 11

1.3: Using Reasoning to Find a Counterexample to a Conjecture

SB: pp.18-26
TR: pp.17-23

Number and Logic

Outcomes

Students will be expected to

NL1 Continued ...
Achievement Indicator:

NL1.4 *Continued*

Elaborations—Strategies for Learning and Teaching

Students could investigate the following examples:

(i) Melissa made a conjecture about slicing pizza. She noticed a pattern between the number of slices of pizza and the number of cuts made in the pizza.

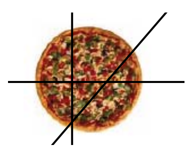
Number of cuts	0	1	2
Number of slices of pizza	1	2	4

Her conjecture was that the number of pizza slices doubled with each cut. Do you agree or disagree? Justify your decision.

As teachers observe students work, they should question students about the considerations that have to be taken into account. Prompt discussion using the following questions:

- Is it important to take into account how the pizza is cut?
- Do all the slices of pizza have to be the same?

A possible counterexample for this situation is shown below:



(ii) Allie creates a series of rectangles, each having different dimensions.

Length (cm)	Width (cm)	Area (cm^2)	Perimeter (cm)
5	4	20	18
10	3	30	26
6	6	36	24
8	3	24	22

Allie makes the conjecture that the area of a rectangle is always greater than its perimeter. Do you agree with this conjecture? Justify your reasoning.

A counterexample disproving this conjecture would be a rectangle having a length of 6 cm and a width of 1 cm. This would result in an area of 6 cm^2 and a perimeter of 14 cm.

General Outcome: Develop number sense and logical reasoning.

Suggested Assessment Strategies

Performance

- Provide students with cards containing a conjecture that may be disproven using counterexamples and have them complete a *Quiz-Quiz-Trade* activity. Student 1 must read the conjecture on the card and student 2 must provide a counterexample to disprove the statement. When both students have disproven the conjectures, they exchange cards and find a new partner. Examples of conjectures include:

  (i) If a number is divisible by 2, then it is divisible by 4.

  (ii) No triangles have two sides of the same length.

  (iii) All basketball players are more than 6 feet tall.

  (iv) If it is a cell phone, then it has a touch screen.

(NL1.4)

Resources/Notes

Authorized Resource

Principles of Mathematics 11

1.3: Using Reasoning to Find a Counterexample to a Conjecture

SB: pp.18-26
TR: pp.17-23

Number and Logic

Outcomes

Students will be expected to

NL1 Continued ...

Achievement Indicators:

NL1.5 *Prove a conjecture, using deductive reasoning (not limited to two column proofs).*

NL1.6 *Prove algebraic and number relationships such as divisibility rules, number properties, mental mathematics strategies or algebraic number tricks.*

Elaborations—Strategies for Learning and Teaching

Deductive reasoning involves drawing a specific conclusion through logical reasoning by starting with general statements that are known to be valid. In this unit, students will be introduced to geometric proofs using deductive reasoning. There will be a much greater emphasis, however, placed on proofs in the next unit.

When constructing proofs, it is necessary for students to use proper mathematical terminology. Students have been exposed to the Pythagorean theorem, the number system, and divisibility rules from previous grades. There are some concepts, however, that will need to be introduced such as supplementary angles, complementary angles, and the transitive property. Students have been exposed to the term straight angle and if angles a, b and c make a straight angle then $a + b + c = 180°$. They are not familiar, however, with the term supplementary. These terms will be reinforced again in the next unit.

Students could begin their exploration of proof by looking at examples involving the transitive property. Consider the following example:

All natural numbers are whole numbers.
All whole numbers are integers.
3 is a natural number.
What can be deduced about the number 3?

Students should be exposed to the different strategies they can use to prove conjectures.

- Students can use a visual representation, such as a Venn diagram, to help them understand the information in an example.
- They can express conjectures as general statements. This involves choosing a variable to algebraically represent the situation. For example, two consecutive integers can be represented as x and $x+1$.
- Students can use a two-column proof, containing statements and reasons. As an alternative, the steps used to prove the conjecture can be communicated in paragraph form.

General Outcome: Develop number sense and logical reasoning.

Suggested Assessment Strategies

Paper and Pencil

- Ask students to prove the following conjectures deductively:

  (i) Mammals have fur (or hair). Lions are classified as mammals. What can be deduced about lions?

  (NL1.5)

  (ii) Prove that the sum of a 2-digit number and the number formed by reversing its digits will always be divisible by 11.

  (NL1.5, NL1.6)

  (iii) Prove that the difference between an odd integer and an even integer is odd.

  (NL1.5, NL1.6))

Performance

- Ask students to create a number trick that always results in a final answer of 4. Prove the conjecture deductively.

  (NL1.5, NL1.6)

Observation

- Ask students to work in groups of three for this activity. Provide each group with a conjecture, such as those shown below, and a strategy to prove the conjecture.

  (i) Each base angle of a right isosceles triangle equals 45°.

  (ii) The value n is a factor of x. 2 is a factor of n. Therefore, 2 is a factor of x.

  (iii) When two lines intersect, the opposite angles are equal.

  Teachers should observe the groups and ask questions such as the following:

  (i) Did you find your strategy difficult for your conjecture? Why or why not?

  (ii) Could you use another strategy? If so, which one? Why?

  (iii) Is one strategy more efficient than another?

  (NL1.5, NL1.6)

Resources/Notes

Authorized Resource

Principles of Mathematics 11

1.4: Proving Conjectures: Deductive Reasoning

SB: pp.27-35

TR: pp.24-29

Number and Logic

Outcomes

Students will be expected to

NL1 Continued ...

Achievement Indicators:

NL1.5, NL1.6 *Continued*

Elaborations—Strategies for Learning and Teaching

It is important for students to recognize that certain strategies are more efficient than others depending on the problem. This is an opportunity for students to work in groups to investigate a number of proofs given a specific method. After students have completed the activity, pose the following questions to promote class discussion:

- What strategy was used to prove your conjecture?
- Could you use another strategy to prove the conjecture?
- Is one strategy more efficient than another? Why?

Students should recognize when proving a conjecture involving a geometric figure, for example, a two-column proof or paragraph form is most appropriate. To prove a conjecture involving number properties, divisibility rules or algebraic number tricks, expressing conjectures as general statements is often the most appropriate method.

When constructing proofs, including algebraic and number relationships, a common error occurs when students use a numerical value as their example. Students need to understand that a conjecture is only proved when all cases are shown to be true. Consider the following example:

- Choose a number. Add 5. Double the result. Subtract 4. Divide the result by 2. Subtract the number you started with. The result is 3.

Students can show inductively, using three or four numbers, that the result is always three. To prove deductively, students must show this is the case for all numbers by letting the first number be x. This leads into the next indicator that compares inductive and deductive reasoning.

General Outcome: Develop number sense and logical reasoning.

Suggested Assessment Strategies

Paper and Pencil

- Ask students to do the following number tricks and then prove them deductively:

  (i) Fabulous Five:
  - Choose a number from 1 to 10.
  - Double the number.
  - Add 10 to your new number.
  - Now, divide the total by 2.
  - Finally, subtract your original number that you started with.
  - Your answer will always be 5!

  (NL1.5, NL1.6)

  (ii) Is This Your Number?
  - Think of any number and write it on a small sheet of paper. Fold it up and place it in your pocket. Remember this number.
  - Find a partner and ask them to think of a number. Make sure they don't tell you.
  - Tell them to double the number they chose.
  - Now this is where your number on the paper comes in. In your head, double the number you wrote. Tell your partner to add the number you doubled in your head.
  - Then tell them to divide this new number by 2.
  - Then tell them to subtract the number they currently have from the first one they started with, or vice versa.
  - Pull out the piece of paper in your pocket and give it to your partner. They will be amazed because the number on the paper will be their number!

  (NL1.5, NL1.6)

Portfolio

- Using the Internet, ask students to search for an example of a number trick and prove it deductively.

  (NL1.5, NL1.6)

Resources/Notes

Authorized Resource

Principles of Mathematics 11

1.4: Proving Conjectures: Deductive Reasoning

SB: pp.27-35
TR: pp.24-29

Number and Logic

Outcomes

Students will be expected to

NL1 Continued ...

Achievement Indicator:

NL1.7 *Compare, using examples, inductive and deductive reasoning.*

Elaborations—Strategies for Learning and Teaching

It is important for students to compare inductive and deductive reasoning through the use of examples. The example below could be used to highlight their differences.

- The sum of four consecutive integers is equal to the sum of the first and last integers multiplied by two.

Inductive Reasoning	Deductive Reasoning
1, 2, 3, 4 1+2+3+4=10 2(1+4)=10	let the numbers be x, $x+1$, $x+2$, $x+3$ $x + (x+1) + (x+2) + (x+3) = 4x+6$ $2[x + (x+3)] = 2(2x + 3) = 4x+6$

Exploring the differences between inductive and deductive reasoning through examples will strengthen student understanding of these concepts. The table below could be used to summarize inductive reasoning versus deductive reasoning.

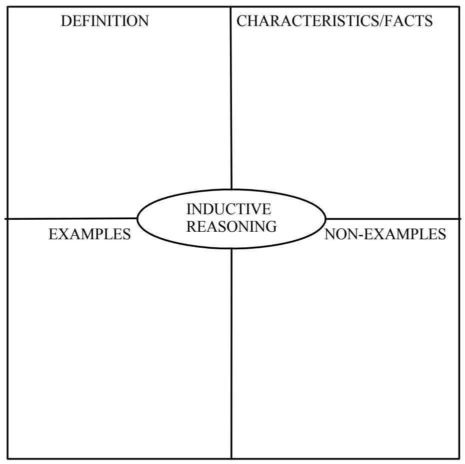
Inductive Reasoning	Deductive Reasoning
Begins with experiences or a number of observations.	Begins with statements, laws, or rules that are considered true.
An assumption is made that the pattern or trend will continue. The result is a conjecture.	The result is a conclusion reached from previously known facts.
Conjectures may or may not be true. One counterexample proves the conjecture false.	Conclusion must be true if all previous statements are true.
Used to make educated guesses based on observations and patterns.	Used to draw conclusions that logically flow from the hypothesis.

General Outcome: Develop number sense and logical reasoning.

Suggested Assessment Strategies

Paper and Pencil

- Ask students to complete the Frayer Model to represent inductive reasoning.



(NL1.7)

Journal

- Ask students to identify situations in their everyday lives where they use inductive and deductive reasoning. They should provide examples of each.

(NL1.7)

Presentation

- Ask students to watch a video clip of a court scene such as one from *A Few Good Men, CSI, Lincoln Lawyer* or another similar show/movie. Ask them to identify how inductive/deductive reasoning is used.

(NL1.7)

Interview

- Brian conjectured that adding two consecutive odd numbers will always equal an even number (i.e., 3 + 5 = 8, 5 + 7 = 12, and 17 + 19 = 36). Did he use inductive or deductive reasoning? Explain your reasoning.

(NL1.7)

Performance

- Ask students to work in groups for the following activity. Each group will be given a deck of cards with a variety of conjectures on them. Students will work together to determine which use inductive reasoning and which use deductive reasoning.

(NL1.7)

Resources/Notes

Authorized Resource

Principles of Mathematics 11

1.4: Proving Conjectures: Deductive Reasoning

SB: pp.27-35
TR: pp.24-29

Number and Logic

Outcomes

Students will be expected to

NL1 Continued ...

Achievement Indicator:

NL1.8 *Identify errors in a given proof; e.g., a proof that ends with 2 = 1.*

Elaborations—Strategies for Learning and Teaching

It is beneficial to have students analyze proofs that contain errors. To reinforce students' understanding of inductive and deductive reasoning, they should identify errors in a given proof, explain why those errors might have occurred and how they can be corrected. Some typical errors include the following:

- Proofs that begin with a false statement:

  All high school students like Facebook. Rebecca is a high school student. Therefore, Rebecca likes Facebook.

Gathering a large quantity of data will strengthen the validity of the conjecture, but to prove a conjecture all cases must be considered. Students should consider if it is possible to ask all high school students if they like Facebook.

- Algebraic errors

Shelby was trying to prove this number trick:	Shelby wrote the following:
• pick a number	Let n be your number
• double your number	$2n$
• add 20	$2n + 20$
• divide by 2	$n + 20$
• subtract the original number	$n + 20 - n$
• the result is 10	20

  Students should identify and correct the error in Shelby's work.

- Division by zero

  Pedro claims he can prove that 2 = 5. His work is shown below:
  Suppose a = b

$-3a = -3b$	Step 1: Multiplying by -3
$-3a + 5a = -3b + 5a$	Step 2: Add $5a$ to both sides
$2a = -3b + 5a$	Step 3: Simplify
$2a - 2b = -3b + 5a - 2b$	Step 4: Subtract $2b$ from each side
$2a - 2b = 5a - 5b$	Step 5: Simplify
$2(a - b) = 5(a - b)$	Step 6: Factor
$\frac{2(a-b)}{(a-b)} = \frac{5(a-b)}{(a-b)}$	Step 7: Divide by $(a - b)$
$2 = 5$	

This type of question may be more challenging for students because algebraically there does not appear to be any mistakes in Pedro's work.

General Outcome: Develop number sense and logical reasoning.

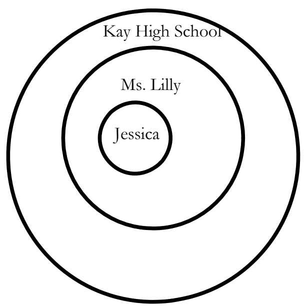
Suggested Assessment Strategies

Paper and Pencil

- Richard was given the following situation:

  Ms. Lilly is a teacher at Kay High School. Jessica is a student at Kay High School. Ask students to write a conjecture about the relationship between Ms.Lilly and Jessica.

  Richard represented his conclusion in the Venn Diagram below and concluded that Jessica is a student of Ms. Lilly's. Ask students to identify Richard's error and construct a new diagram to represent this situation.



(NL1.8)

Performance

- Create centres in the classroom containing proofs which contain errors. Ask students to participate in a carousel activity where they move throughout the centres to identify and correct the error.

(NL1.8)

Resources/Notes

Authorized Resource

Principles of Mathematics 11

1.5: Proofs That Are Not Valid

SB: pp.36-44
TR: pp.30-34

Number and Logic

Outcomes

Students will be expected to

NL1 Continued ...

Achievement Indicators:

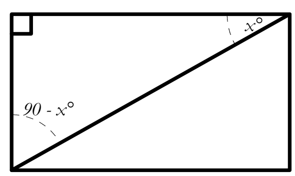
NL1.8 *Continued*

Elaborations—Strategies for Learning and Teaching

Students should analyze Step 6 and Step 7 carefully. In Step 6, students may use substitution instead of division. Since $a - b = 0$, students may write $2(a - b) = 5(a - b)$ as $2(0) = 5(0)$ resulting in $0 = 0$. In Step 7, students may write $2(a - b) = 5(a - b)$, as $2 = 5$ without realizing they have even divided. Ask students to substitute $a = b$ back into the equation. When working with proofs that involve division, students should check to see if the divisor is zero.

- Circular reasoning

An argument is circular if its conclusion is among its premises. Darren claims he can prove that the sum of the interior angles in a triangle is 180°.



Here is his proof: I constructed a rectangle. Next, I drew a diagonal. I knew that all of the angles in a rectangle are 90°. I labelled one of the other angles in the triangle x. Therefore, the other angle must be $180° - 90° - x = 90° - x$. Then, $90° + x + (90° - x) = 180°$.

Students must realize that they can't assume a result that follows from what they are trying to prove.

NL1.9 *Solve a contextual problem that involves inductive or deductive reasoning.*

Students should be exposed to problem solving situations that require the use of inductive and/or deductive reasoning. They will explore some situations where they are asked to first show inductively that a pattern exists and then prove it deductively. It is important for students to recognize that inductive and deductive reasoning are not separate entities, they work together. Consider the following example:

Tyler was investigating patterns on the hundreds chart. He was asked to choose any four numbers that form a 2 x 2 square on the chart. He chose the following:

4	5
14	15

He should be able to use inductive reasoning to make a conjecture about the sum of each diagonal and then use deductive reasoning to prove his conjecture is always true.

General Outcome: Develop number sense and logical reasoning.

Suggested Assessment Strategies

Paper and Pencil

- Ten men meet for a bowling tournament and each shakes the hand of every other man. Ask students to determine the number of handshakes that occurred. They should explain the strategy used to arrive at the answer.

(NL1.9)

- Look at a monthly calendar and pick any 3 squares in a row- across, down or diagonal. Using inductive reasoning, ask students to make a conjecture about the middle number. They should then use deductive reasoning to prove their conjecture.

(NL1.9)

Observation

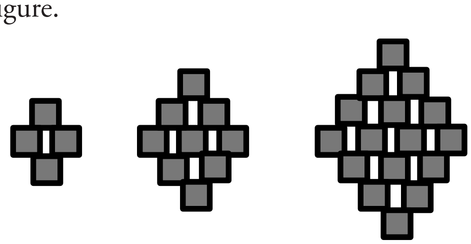
- Have students engage in an activity where they move from station to station to solve various problems. Problems could include the following:

  (i) If you have a 5-L and 3-L bottle and plenty of water, how can you get 4-L of water in the 5-L bottle?

  (ii) Ted, Ken, Allyson, and Janie (two married couples) each have a favourite sport: swimming, running, biking, and golf. Given the following clues, determine who likes which sport.

  - Ted dislikes golf.
  - Each woman's favourite sport is featured in a triathlon.
  - Ken nor his wife enjoy running.
  - Allyson bought her husband a new bike for his birthday to use in his favourite sport.

  (iii) Determine the number of squares that would be in the next (4th) figure.



(NL1.9)

Performance

- Ask students to participate in a game of *Clue.* They should explain how the game uses inductive and/or deductive reasoning.

(NL1.9)

Resources/Notes

Authorized Resource

Principles of Mathematics 11

1.5: Proofs That Are Not Valid

SB: pp.36-44
TR: pp.30-34

1.6: Reasoning to Solve Problems

SB: pp.45-51
TR: pp.35-38

Web Link

The iPhone/iPod/iPad app called *Crack the Code* is an application involving deductive and/or inductive reasoning.

Number and Logic

Outcomes

Students will be expected to

NL1 Continued ...

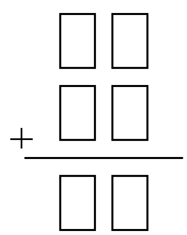
Achievement Indicator:

NL1.9 *Continued*

Elaborations—Strategies for Learning and Teaching

Encourage students to participate in the following activity to recognize patterns and to try and solve puzzles.

Each group will need a set of digit cards (two or three copies of the digits 0-9) and a template of the algorithm. A sample is shown below where the algorithm is a two column sum.



Find The Sum	
Adding the digits in the top number gives 10.	All of the digits are even numbers.
Adding the digits in the second number gives 4.	The digit 0 is not used in any number.
The difference between the digits in the answer is 2.	The ones digit in the top number is the same as the tens digit in the answer.

Prompt students discussion with the following questions:

- How did you start the problem?
- Did you notice any patterns?
- What were your initial numbers?
- How could you convince someone that you have identified the solution?

General Outcome: Develop number sense and logical reasoning.

Suggested Assessment Strategies

Observation

- Ask students to complete the following puzzle. Create cards which contain pieces of a puzzle. Teachers can distribute different puzzles or the same puzzle with a different title. Each student is given a card and must find their group based on the title of the puzzle listed on the card. To maintain 'ownership' of the piece of information, the student may not physically give away the clue-card, but must be responsible for communicating the content to the group. Each student's role is to work within his/her group to solve the puzzle, following the set of work rules. It is important for groups to report on their problem-solving processes as well as confirming the correctness of their end product.

  A sample puzzle is shown below. It may be helpful if members of the group were provided a hundreds chart.

Peter's Number	
Peter's number is a multiple of four.	Peter's number is in the bottom half of the chart.
One of the digits is NOT double the other digit.	When you add the digits, you get an odd number.
Peter's number is a multiple of six.	Peter's number is even.

  The teacher can focus on the following questions while observing the activity:

  (i) How did your group get started?

  (ii) How did the group 'join' the clues together?

  (iii) Were there any challenges?

  (iv) Which clue did you find most helpful?

  (v) What might you do differently next time?

(NL1.9)

Resources/Notes

Authorized Resource

Principles of Mathematics 11

1.6: Reasoning to Solve Problems

SB: pp.45-51
TR: pp.35-38

Number and Logic

Outcomes

Students will be expected to

NL2 Analyze puzzles and games that involve spatial reasoning, using problem-solving strategies.

[CN, PS, R, V]

Achievement Indicator:

NL2.1 *Determine, explain and verify a strategy to solve a puzzle or to win a game; e.g.,*

- *guess and check*
- *look for a pattern*
- *make a systematic list*
- *draw or model*
- *eliminate possibilities*
- *simplify the original problem*
- *work backward*
- *develop alternative approaches*

Elaborations—Strategies for Learning and Teaching

This outcome is intended to be integrated throughout the course by using puzzles and games focused on sliding, rotation, construction and deconstruction. They are intended to help enhance spatial reasoning and problem-solving strategies. Numerical reasoning and logical reasoning using puzzles and games will be addressed in Mathematics 3201. Students need time to play and enjoy the game before analysis begins. They can then discuss the game, determine the winning strategies and explain these strategies by demonstration, orally or in writing.

A variety of puzzles and games, such as board games, online puzzles and games, appropriate selections for gaming systems and pencil and paper games should be used. The puzzles and games may relate directly to other outcomes in this course, but the main purpose is to develop spatial reasoning.

It is not intended that the activities be taught in a block of time, but rather explored periodically during the year.

Games give students a chance to develop positive attitudes towards mathematics by reducing the fear of failure and error. Games provide students with an opportunity to interact with other students, to explore intuitive ideas and problem-solving strategies. Students' thinking often becomes apparent through the actions and decisions they make during a game, so teachers have the opportunity to formatively assess learning in a non-threatening situation.

Problem-solving strategies will vary depending on the puzzle or game. Some students will explain their strategy by working backwards, looking for a pattern, using guess and check, or by eliminating possibilities, while others will plot their moves by trying to anticipate their opponents' moves. As students play games and analyze strategies, they explore mathematical ideas and compare different strategies for efficiency.

The following are some tips for using games in the mathematics class:

- Use games for specific purposes.
- Keep the number of players from two to four, so that turns come around quickly.
- Communicate to students the purpose of the game.
- Engage students in post-game discussions.

General Outcome: Develop number sense and logical reasoning.

Suggested Assessment Strategies

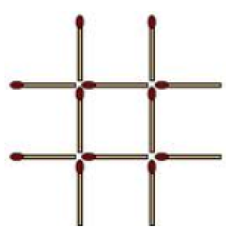
Discussion

- As students play games or solve puzzles, ask probing questions and listen to their responses. Record the different strategies and use these strategies to begin a class discussion. Possible discussion starters include:
  - (i) Thumbs up if you liked the game, thumbs sideways if it was okay, and thumbs down if you didn't like it. What did you like about it? Why?
  - (ii) What did you notice while playing the game?
  - (iii) Did you make any choices while playing?
  - (iv) Did anyone figure out a way to quickly find a solution?

(NL2.1)

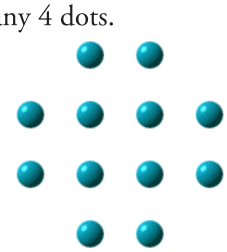
Performance

- Ask students to participate in the following activities. They should discuss the strategies used and identify any challenges that occurred while trying to solve the puzzle.
  - (i) Move 3 matchsticks to make three identical squares.



(NL2.1)

  - (ii) How many squares can you create in this figure by connecting any 4 dots.



(NL2.1)

Resources/Notes

Authorized Resource

Principles of Mathematics 11

1.7: Analyzing Puzzles and Games

SB: pp.52-57
TR: pp.39-42

Number and Logic

Outcomes

Students will be expected to

NL2 Continued...

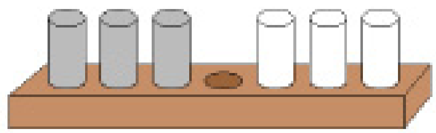
Achievement Indicator:

NL2.1 *Continued*

Elaborations—Strategies for Learning and Teaching

As students play a game, it is important to pose questions and engage students in discussions about the strategies they are using.

Consider the following peg game. The goal of the puzzle is to switch the pegs on the left with the pegs on the right by moving one peg at a time. A peg may only be moved to an open slot directly in front of it or by jumping over a peg to an open slot on the other side of it. You may not move backwards. The game ends when you win or get stuck.



As teachers observe students playing the game, ask them questions such as the following:

- What did you like about the game? Why?
- What did you notice while playing?
- What was your first move?
- What were your first three moves?
- What problems arose when solving the puzzle?
- What are the minimum number of moves to win this game?
- Did you anticipate the next move?
- Did you notice any patterns?

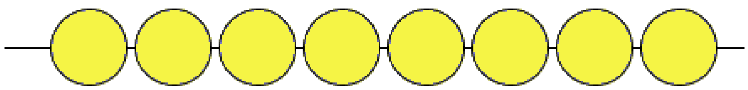
Students should be encouraged to explore a simpler version of the game where appropriate. Solving the peg puzzle using two or four pegs, for example, may allow students to complete the game more efficiently.

General Outcome: Develop number sense and logical reasoning.

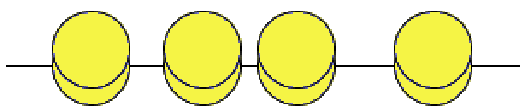
Suggested Assessment Strategies

Performance

- Ask students to take eight pennies and arrange them in row. The goal is to end up with four piles of two pennies each. Each move consists of picking up a penny, jumping in either direction over two "piles" and landing on the third. Each pile may consist of a single coin or two coins. This must be done in four moves. Ask them to discuss the strategy used to solve the puzzle.

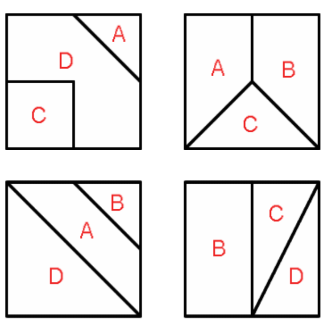


Goal:



(NL2.1)

- The clues in this type of problem are non-verbal. In this activity, the squares are cut up by the teacher and each member of a group-of-four is given three pieces marked with the same letter. Ask the group to make four complete squares.



(NL2.1)

Resources/Notes

Authorized Resource

Principles of Mathematics 11

1.7: Analyzing Puzzles and Games

SB: pp.52-57
TR: pp.39-42

Web Link

Sliding coin puzzles can be found at: *http://www.alethis.net*

The interactive peg game and other interactive games can be found at: *http://nlvm.usu.edu (Keyword: Select 9-12)*

Number and Logic

Outcomes

Students will be expected to

NL2 Continued...

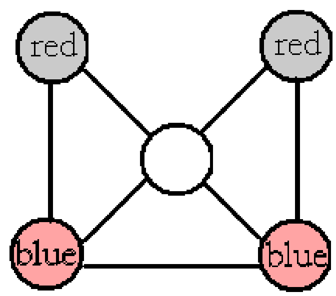
Achievement Indicators:

NL2.1 *Continued*

NL2.2 *Create a variation on a puzzle or a game, and describe a strategy for solving the puzzle or winning the game.*

Elaborations—Strategies for Learning and Teaching

An example of a board game that resembles Tic-Tac-Toe is Pong Hau K'i, which is also played by two players. The game board is made up of five circles joined by seven line segments. Students will need to draw a game board. Each player will have two stones (counters or buttons) of different colours. Place two red stones at the top, for example, and two blue at the bottom as shown below.



Players take turns sliding one stone along a line to an adjacent empty circle. To win, you have to block the other player so that he or she cannot move.

Students should play the game several times, taking turns making the first move. Ask the following questions as they play the game:

- Where will the first move always be?
- What does the board look like when one player is blocked? Why will this help the player?
- Is it better to have the red stones or blue stones?
- Is it better to make the first move or second move?
- Is anticipating the possible moves ahead an effective game plan strategy? Explain why or why not.

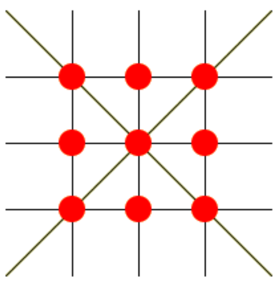
A variation of this game is to start from different positions. Decide which colour each player will use and who will place the first stone. After the first stone is placed on any circle, the other player places a stone on any of the four remaining circles and so on until all four stones are on the board. The game is continued by moving the pieces as previously decribed. Ask students to play the modified game several times. Is it more challenging than the original game?

General Outcome: Develop number sense and logical reasoning.

Suggested Assessment Strategies

Performance

- Take nine pennies and arrange them in a 3 by 3 grid. There are now eight straight lines that each pass through three pennies: three horizontal, three vertical and two diagonal. Your goal is to move exactly two pennies so that there are ten straight lines, each with three pennies. In the solution, there are no straight lines with more than three pennies, and no penny is placed on top of another.



(NL2.1)

Resources/Notes

Authorized Resource

Principles of Mathematics 11
1.7: Analyzing Puzzles and Games

SB: pp.52-57
TR: pp.39-42

Web Link

An interactive game of Pong Hau K'i can be found at:
http://nrich.maths.org
(Keyword: Pong)

Mathematics Inquiry

Suggested Time: 6 Hours

Unit Overview

Focus and Context

In this unit, students will research and give a presentation on a historical or current event, or an area of interest that involves mathematics. They will make connections between ideas and choices about which aspects of the project they should investigate.

Students will decide on their topic and conduct research. If the research involves a collection of data, they can use statistical tools to analyze and interpret the data. The statistical reasoning unit, is done later in this course. Students will have to take this into consideration when then do their planning for their project.

The outline of the research project is presented in this unit to provide students with an overview of the skills, time and resources needed. They should be presented with reasonable timelines for each stage of student work.

Outcomes Framework

GCO

Develop an appreciation of the role of mathematics in society.

SCO RP1

Research and give a presentation on a historical or current event, or an area of interest that involves mathematics.

Process Standards Key

[C]	Communication	**[PS]**	Problem Solving
[CN]	Connections	**[R]**	Reasoning
[ME]	Mental Mathematics and Estimation	**[T]**	Technology
		[V]	Visualization

SCO Continuum

Mathematics 1201	Mathematics 2201	Mathematics 3201
	Mathematics Research Project	
	RP1 Research and give a presentation on a historical or current event, or an area of interest that involves mathematics. [C, CN, ME, PS, R, T, V]	

Mathematics Research Project

Outcomes

Students will be expected to

RP1 Research and give a presentation on a historical or current event, or an area of interest that involves mathematics.

[C, CN, ME, PS, R, T, V]

Elaborations—Strategies for Learning and Teaching

Projects give students opportunities to build on skills such as critical thinking, problem solving, communication, collaboration, and technology literacy. The goal is for the project to be engaging, interesting and relevant to the student.

Teachers should use the following guidelines when project-based learning is used:

- Create teams to work on the project or students may work independently.
- Allow students choice on the topic and on the project presentation.
- Calendar the project through plans, drafts and timely benchmarks.
- Design the assessment plan in advance and inform students of the criteria before they start their project. This will help them with their progress and direct their own learning.
- Act as a facilitator rather than a leader.
- Choose to begin a stage of the project at the beginning/end of each chapter.

It is critical to give students class time to complete some of the steps necessary, such as brainstorming, writing an outline, analyzing data, making decisions and using technology. The time will be most productive if, at the end of each class work session, tasks are assigned to be completed prior to the next meeting.

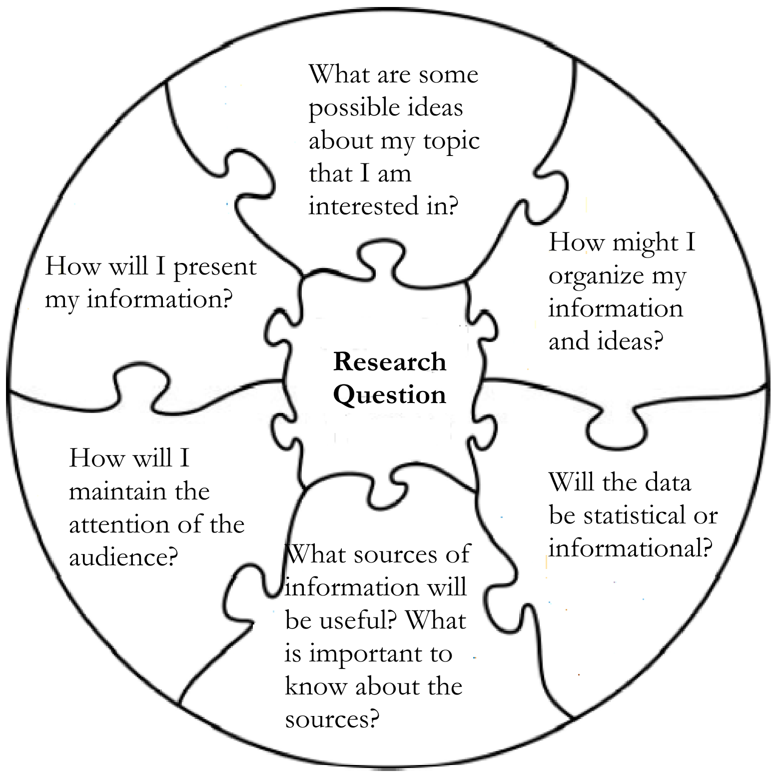
A rubric should be used to assess the project and students should be aware of the criteria before they start their project. Encourage them to participate in the development of the rubric and to work out the approriate categories and criteria for specific tasks. This will improve their motivation, interest, and performance in the project. Content, organization, sources and layout are critical components used to evaluate projects. Illustrations, images and graphics are also important features that should be included. Remind students that there are many different ways to deliver a project. Therefore, the rubric may have to be modified to fit the format of the presentation.

General Outcome: Develop an appreciation of the role of mathematics in society.

Suggested Assessment Strategies

Discussion

- Ask students to brainstorm possible questions, ideas and/or issues relating to their topic.



- Ask students to discuss categories and criteria that may be included in a rubric. Suggestions are shown below:

	Excellent	Good	Competent	Needs Improvement
Reasoning	Justifies all mathematical statements in an efficient and accurate manner, and draws valid conclusions.	Justifies most mathematical statements accurately, and draws valid conclusions.	Justifies some of the mathematical statements accurately, and draws valid conclusions.	Does not justify mathematical statements accurately, and does not draw valid conclusions.
Connections	Discusses, in depth, how mathematical concepts interconnect and build on each other.	Discusses how math concepts interconnect and build on each other.	Discusses superficially how math concepts interconnect and build on each other.	Does not discuss the inter-connection between concepts.

Resources/Notes

Authorized Resource

Principles of Mathematics 11

At the end of each chapter, support is provided to help students meet the project outcome. The details are organized under the title Project Connection (PC).

Mathematics Research Project

Outcomes

Students will be expected to

RP1 Continued ...

Achievement Indicators:

RP1.1 *Collect primary or secondary data (statistical or informational) related to the topic.*

RP1.2 *Assess the accuracy, reliability and relevance of the primary or secondary data collected by:*

- *identifying examples of bias and points of view*
- *identifying and describing the data collection methods*
- *determining if the data is relevant*
- *determining if the data is consistent with information obtained from other sources on the same topic.*

RP1.3 *Interpret data, using statistical methods if applicable.*

RP1.4 *Identify controversial issues, if any, and present multiple sides of the issues with supporting data.*

RP1.5 *Organize and present the research project, with or without technology.*

Elaborations—Strategies for Learning and Teaching

Encourage students to create and submit an action plan or graphic organizer outlining their tasks and timeframes. For example, a project based on primary data (data that students collect) will usually require more time than a project based on secondary data (data that other people have collected and published).

Depending on the research question, teachers should discuss the different stages that may be involved in this project. They can use the following outline as a guide to promote students discussion:

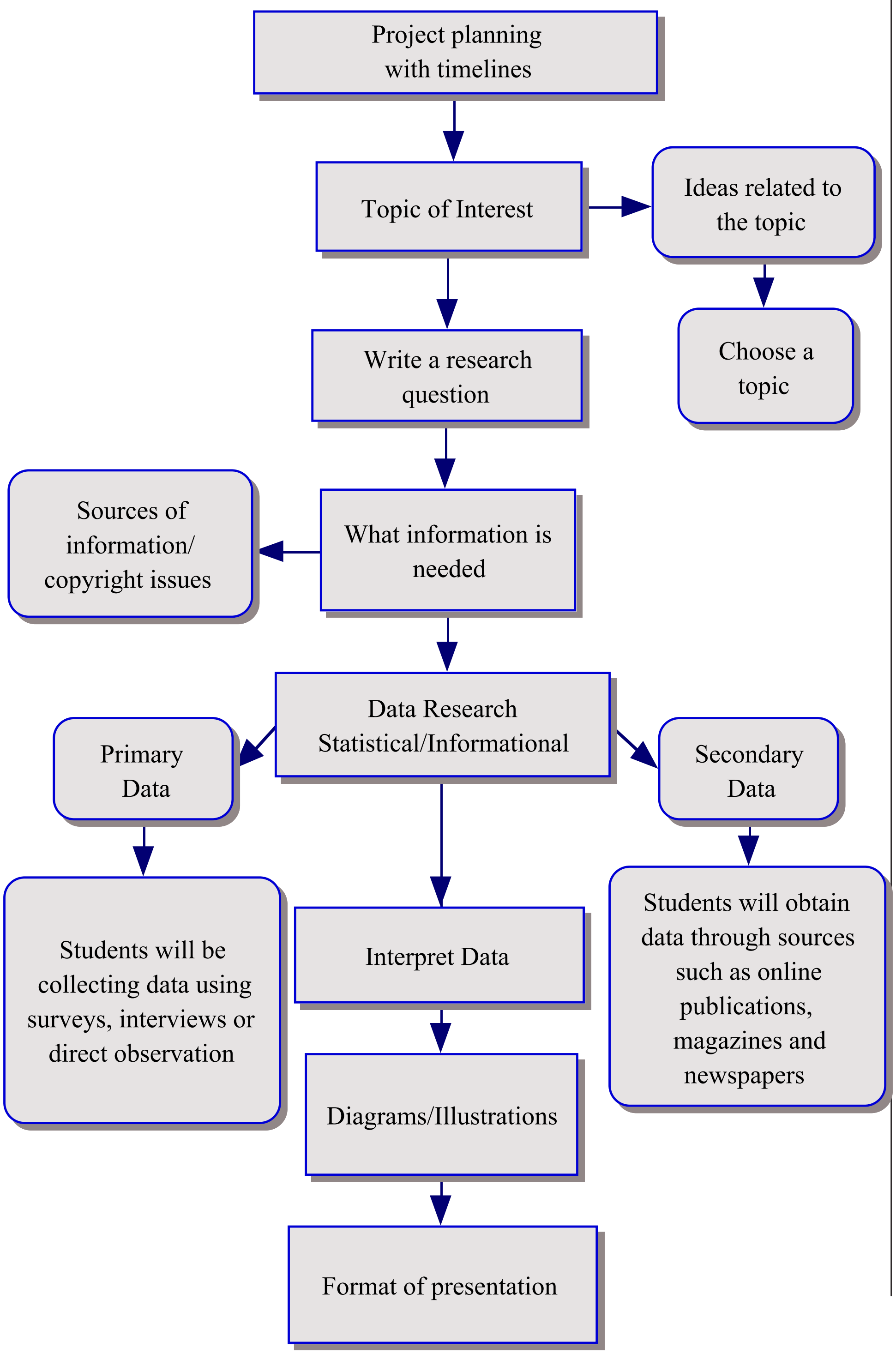
- Ask students to select a topic of interest and create a research question. Encourage them to use a concept map to organize their thoughts and ideas.
- Ask students if their topic is practical to research.
- Is their research based more on informational data or statistical data? Will they collect data and what plan will they use to collect it? Will they use data already collected? Is the data current and reliable?
- What does the information mean? What are there conclusions?
- If the data is statistical in nature, what tools will they use to analyze and interpret the data?
- Ask students about controversial issues they may uncover during their research. Provide students with the following example. People may have an opinion that low marks are due to playing too many video games. The student should determine whether there is reasonable support for the claims being made or if there are other factors to consider.
- Ask students about the presentation of the project. To make the presentation interesting, encourage students to choose a format that suits the student's style. Will students use a multimedia presentation such as a video, or a power point? Will it be in the form of a podcast? Will students create their own website or facebook page to present their project? Some students may prefer to present without the use of technology and use, for example, a poster board, a pamphlet, a debate, an advertising campaign or a demonstration.

General Outcome: Develop an appreciation of the role of mathematics in society.

Suggested Assessment Strategies

Graphic Organizer

- Ask students to submit a graphic organizer outlining their plan and timelines. A sample is provided below:



(RP1.1, RP1.2, RP1.3, RP1.4, RP1.5)

Resources/Notes

Authorized Resource

Principles of Mathematics 11

PC 1: Creating an Action Plan
SB: pp.64-65
TR: p.49

Web Links

The following tools are available to create projects.

A cloud-based presentation software that is visually captivating can be found at: *www.prezi.com*

A great web application that allows students to brainstorm online and create mind maps can be found at *www.bubbl.us.*

An outlet for creative posters or glogs can be found at. *www.edu.glogster.com*

The following is a web application that produces videos from user-selected photos, videoclips and music: *www.animoto.com*

An online photo editing tool can be found at: *www.funphotobox.com*

Mathematics Research Project

Outcomes

Students will be expected to

RP1 Continued ...

Achievement Indicators:

RP1.1, RP1.2

RP1.3, RP1.4

RP1.5 *Continued*

Elaborations—Strategies for Learning and Teaching

Students should have the option to choose a topic from a list provided by the teacher or choose a topic that relates directly to something of interest to them. The topic does not necessarily have to relate to something studied in this course. Provide the following suggested topics:

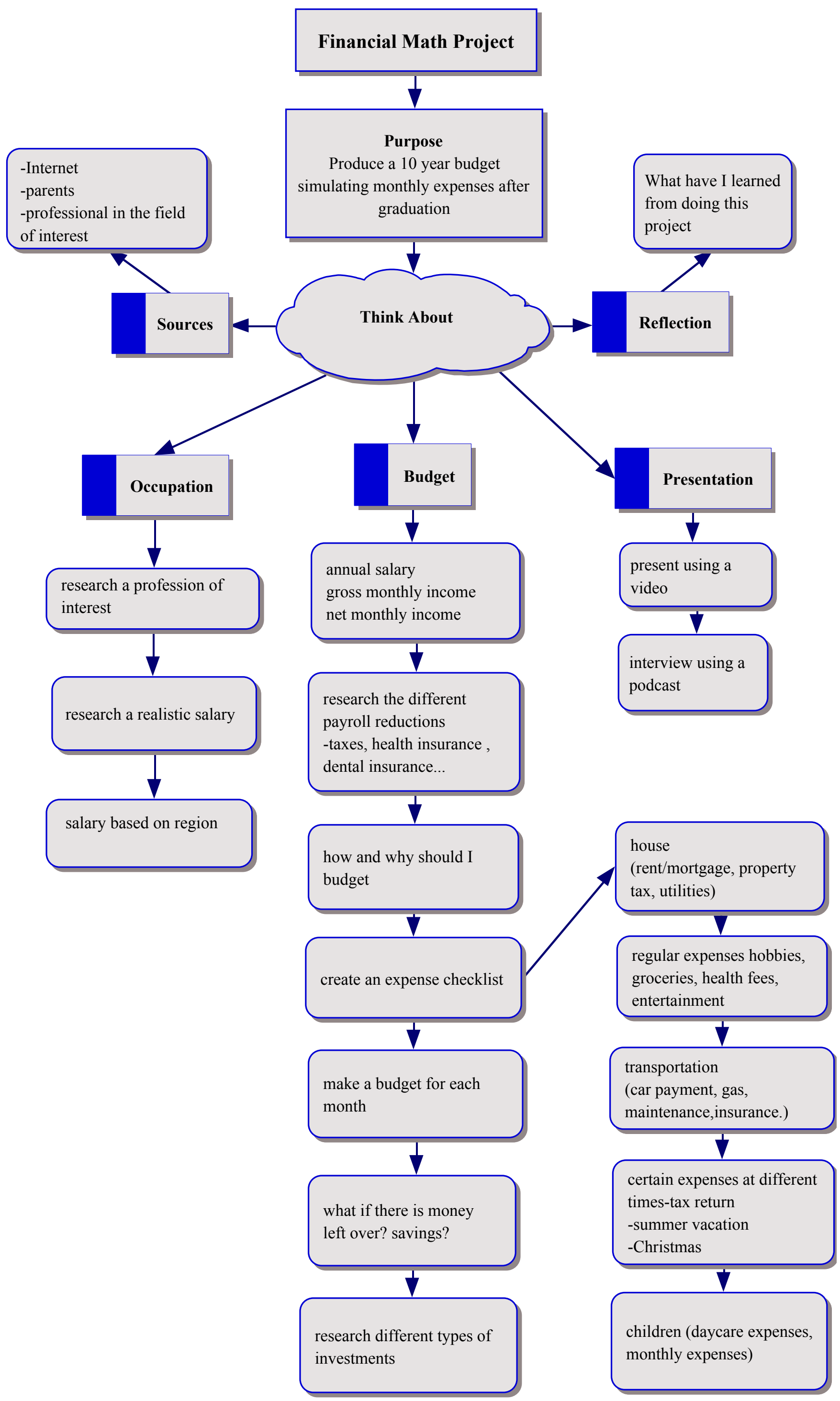
- Ask students to research surveying. Find out and report on how surveyors use trigonometry in their work.
- Ask students to discuss the history and effectiveness of a defibrillator. (i.e., They could use historical and/or current data to differentiate between its effectiveness as it applies to a patient's age, sex, family history, etc.)
- Ask students to research the different designs of skateboarding ramps and the safety regulations when designing a ramp. Design a blueprint of a skateboard ramp and then use a self-chosen scale factor to construct the actual ramp.
- Ask students to research how quadratic functions are used in Robotics.
- Ask students to design a video game where the success of the game is dependent on correctly-answered mathematical questions.
- Ask students to determine which vehicle would be considered their best investment in their region. They should support their findings.
- Ask students to determine which communications package is best for them and their family, based on their region. They should support their findings.
- Ask students to design and model the cost to upgrade a room(s) in their house. This project could be a group competition, analyzing with a given allowance the necessities for a given room on a given budget.
- Ask students to research the payroll deductions and other benefits they would have to consider if they were to start their own business for a set number of employees.
- Ask students to conduct a survey to gather information concerning teen smoking in a school. Research the implications of smoking and create an educational campaign to create awareness of the benefits of non-smoking.
- Ask students to research and design a lifestyle budget based on given characteristics of a typical profession (lawyer, retail, etc.) with varying demographics (urban, rural). Be sure to include health plans, recreational activities, fixed expenses, etc.

General Outcome: Develop an appreciation of the role of mathematics in society.

Suggested Assessment Strategies

Graphic Organizer

- Ask students to create and submit a detailed outline of their project. A sample of a financial plan is provided below.



(RP1.1, RP1.2, RP1.3, RP1.4, RP1.5)

Resources/Notes

Authorized Resource

Principles of Mathematics 11

PC 2: Selecting Your Research Topic
SB: pp.122-123
TR: p.122

PC 3: Creating Your Research Question
SB: pp.170-171
TR: p.178

Mathematics Research Project

Outcomes

Students will be expected to

RP1 Continued ...

Achievement Indicators:

RP1.1, RP1.2

RP1.3, RP1.4

RP1.5 *Continued*

Elaborations—Strategies for Learning and Teaching

- Ask students to research an art work and relate how mathematics was used to create the work. Design an original mural that incorporates similarity, parallel lines, quadratics, or other topics.
- Ask students to research a geometric design or logo and describe the history and significance of it. Replicate the picture incorporating a variety of equations, taking into account their exposure to linear and quadratic equations, along with domain and range. Use a graphing calculator or software programs to draw the sketch.
- Ask students to research a card game to investigate how math is used in creating, playing, and winning the game. (i.e., Students could explore the probability of getting a straight as opposed to a flush in poker.)
- Ask students to research the stockmarket and pick a stock they want to follow for a certain time period. Graph the sales of the stock over several weeks to display how well the stock fared during this time period and research the reasons for fluctuation and whether they should buy or sell.
- Ask students to take the role of an architect and design a shopping mall or fitness centre. Research the design of malls and the feasible sizes for various stores.
- Ask students to take the role of an investment planner. Research various forms of investments and formulas for calculating interest on savings.
- Ask students to take the role of a statistician researching population trends for their province. Discuss the factors that may have played a role in the trends, and predict the population five years from now.
- Ask students to analyze the scoring patterns of several hockey players over their careers. Predict the number of goals they will get in the upcoming season.
- Class decides on careers/trades/local businesses they would like to investigate to see how math is used. Choices might include a plumber, an electrician, a building contractor, dentist, etc. Divide the class into groups and assign one career to each. Ask each group to research how math is used in this career and present findings to the class. Create at least two math problems based on their research.

General Outcome: Develop an appreciation of the role of mathematics in society.

Suggested Assessment Strategies

Performance

- Ask students to research a historical person or event related to mathematics. They could present their project creating a social networking profile. With the project, the students becomes the historical figure and creates a page with the following components:
  - (i) Profile picture
  - (ii) Information corner (personal information and interests)
  - (iii) Other photos of significance
  - (iv) Videos
  - (v) Calendar timeline

  (RP1.1, RP1.2, RP1.3, RP1.4, RP1.5)

Interview

- Ask students questions periodically to ensure that their work is ongoing and to give them support. Observe their attitudes, contributions, time on task, and teamwork. Some possible questions could include:
  - (i) Where are you in the research process?
  - (ii) What are the different roles of your group?
  - (iii) Why did you take that step, use that representation, or try that case?
  - (iv) Are you collecting data or using data already collected?
  - (v) What are you doing next?
  - (vi) Are you experiencing any problems with the research information? Are you having any technological difficulties?
  - (vii) What resources are you using?
  - (viii) How are you presenting your result(s)?
  - (ix) What expectations do you have for your result(s)?
  - (x) What questions do you have?

  (RP1.1, RP1.2, RP1.3, RP1.4, RP1.5)

Resources/Notes

Authorized Resource

Principles of Mathematics 11

PC 2: Selecting Your Research Topic
SB: pp.122-123
TR: p.122

PC 3: Creating Your Research Question
SB: pp.170-171
TR: p.178

Web Links

The following are user friendly sites to create a website: *www.wix.com* or *webs.com* or *weebly.com*

The following is a secure social networking site that can be monitored by teachers: *www.edmodo.com*

Mathematics Research Project

Outcomes

Students will be expected to

RP1 Continued ...

Achievement Indicators:

RP1.1, RP1.2

RP1.3, RP1.4

RP1.5 *Continued*

Elaborations—Strategies for Learning and Teaching

Data collection is divided into two categories, primary and secondary data. Primary data is data that the student collects himself or herself using surveys, interviews or direct observations. Secondary data is data that the student obtains through other sources, such as online publications, journals, magazines, and newspapers. The student must ensure that the source of data is reliable and up to date. Discuss with students the advantages and disadvantages of both types of data. The type of data students will choose will depend on the research question, available time and resources.

Data is a representation of information. It may consist of a collection of descriptions, anecdotes, opinions, and/or interpretations. It may also refer to information that can be translated into numbers which can then be displayed and analyzed mathematically. Detecting patterns and trends is an important part of data analysis. Students may use statistical tools such as measures of central tendency, measures of dispersion and normal distribution to interpret the data. They may represent and organize their data be creating lists, charts, tables, frequency distributions and/or graphs. Depending on their research and research question, students may use a few or many of these tools. As students prepare their timelines, it is important to note the Statistical Reasoning unit is done later in this course.

Selecting which measure of central tendency (mean, median, or mode) to use depends on the distribution of the data. As the researcher, the student must decide which measure most accurately describes the tendencies of the population. Review with students the following criteria to consider when deciding upon which measure of central tendency best describes a set of data:

- If all the data points are relatively close together, the mean gives a good idea as to what the points are closest to.
- Outliers affect the mean the most. If the data includes outliers, the student should use the median to avoid misrepresenting the data.
- If the frequency of the data is more important than the values, it may be beneficial to use the mode.

General Outcome: Develop an appreciation of the role of mathematics in society.

Suggested Assessment Strategies

Discussion

- In small groups, ask students to discuss the importance of collecting, organizing and analyzing data using statistical methods to predict results.

  Note: Discussion could centre around the ability to identify trends related to opinion polls, stock prices, tax rates, crime statistics, scientific studies, or weather reports. They should judge the reasonableness of any claims and interpretations made.

(RP1.1, RP1.2, RP1.3, RP1.4, RP1.5)

Resources/Notes

Authorized Resource

Principles of Mathematics 11

PC 4: Carrying Out Your Research
SB: pp.230-231
TR: p.233

PC 5: Analyzing Your Data
SB: pp.312-313
TR: p.295

Mathematics Research Project

Outcomes

Students will be expected to

RP1 Continued ...

Achievement Indicators:

RP1.1, RP1.2

RP1.3, RP1.4

RP1.5 *Continued*

Elaborations—Strategies for Learning and Teaching

Both the range and the standard deviation will give the student information about the distribution of data in a set, the measures of dispersion. The range of a set of data changes considerably because of outliers. The disadvantage of using the range is that it does not show where most of the data in a set lies, it only shows the spread between the highest and the lowest values. Standard deviation is the measure of dispersion that is most commonly used in statistical analysis. It is the average distance between the actual data and the mean. As students work with standard deviation, data that is more clustered around the mean results in less variation in the data and the standard deviation is lower. If the data is more spread out over a large range of values, there is greater variation and the standard deviation would be higher.

Students can sort data into a frequency distribution table and then progress to creating a histogram and frequency polygon. If the data closely follows a normal distribution, the properties of a normal distribution can be used to analyze it.

It is important for teachers to consider the following options when organizing or planning student presentations.

- *Math Fair:* Student presentations could model a science fair, a heritage fair or any other forum which displays student work. This would be a good opportunity to invite other subject teachers or members of the community to view and/or judge student projects. This structure enables many projects to be judged in a small amount of time.
- *In-Class Presentation*: Students can present their projects to their peers during class time.
- *Video Presentation*: Students can present their projects through video to their peers during class time.

The goal of this project is to give students an opportunity to communicate their learning experience with others and to be creative in their delivery of the presentation.

General Outcome: Develop an appreciation of the role of mathematics in society.

Suggested Assessment Strategies

Observation

- Following is a sample rubric that could be used to assess presentations.

	4	3	2	1
Content	-Includes in-depth information about the topic with details and examples. -Subject knowledge is excellent.	-Includes essential information about the topic. -Subject knowledge is good.	-Includes essential information about the topic but there are 1-2 factual errors. -Subject knowledge is good.	-Content is minimal and/or there are several factual errors. -Subject knowledge is good.
Topic	Demonstrates a strong link between the research topic and mathematics.	Demonstrates a good link between the research topic and mathematics.	Demonstrates some link between the research topic and mathematics.	No link made between the research topic and mathematics.
Organization	Consists of a logical progression of ideas and events.	Consists of a logical progression of ideas and events although minor gaps exist.	Some organization is present but transition between ideas is unclear or does not seem to fit.	There is no clear logical organization structure.
Diagrams/ Illustrations	Visuals used appropriately; greatly enhance the topic and aid in comprehension.	Visuals used appropriately most of the time; most visuals enhance the topic and aid in comprehension.	Visuals used inappropriately and excessiveley; are poorly selected and do not enhance the topic.	Few or no visuals used.
Presentation	Student is able to answer all questions posed and exhibits understanding of the research topic.	Student is able to answer most questions posed and exhibits satisfactory understanding of the research topic.	Student is able to answer some of the questions posed but exhibits some misunderstanding.	Student is unable to answer questions posed and exhibits limited understanding of the research topic.
Sources of Information	Sources meet all 4 of the characteristics: -variety -reliability -relevant -documented	Sources meet 3 of the 4 characteristics: -variety -reliability -relevant -documented	Sources meet 2 of the 4 characteristics: -variety -reliability -relevant -documented	Sources meet 1 of the 4 characteristics: -variety -reliability -relevant -documented

(RP1.1, RP1.2, RP1.3, RP1.4, RP1.5)

Resources/Notes

Authorized Resource

Principles of Mathematics 11

PC 4: Carrying Out Your Research
SB: pp.230-231
TR: p.233

PC 5: Analyzing Your Data
SB: pp.312-313
TR: p.295

PC 6: Identifying Controversial Issues
SB: pp.390-391
TR: p.350

PC 7: The Final Product
SB: pp.438-439
TR: p.416

PC 8: Peer Critiquing
SB: pp.510-511
TR: p.493

Web Link

This website lists rubrics for different types of projects such as Powerpoint, Podcast, Webpage, Video, etc.

www.uwstout.edu.soe/profdev/rubrics.cfm

Properties of Angles and Triangles

Suggested Time: 13 Hours

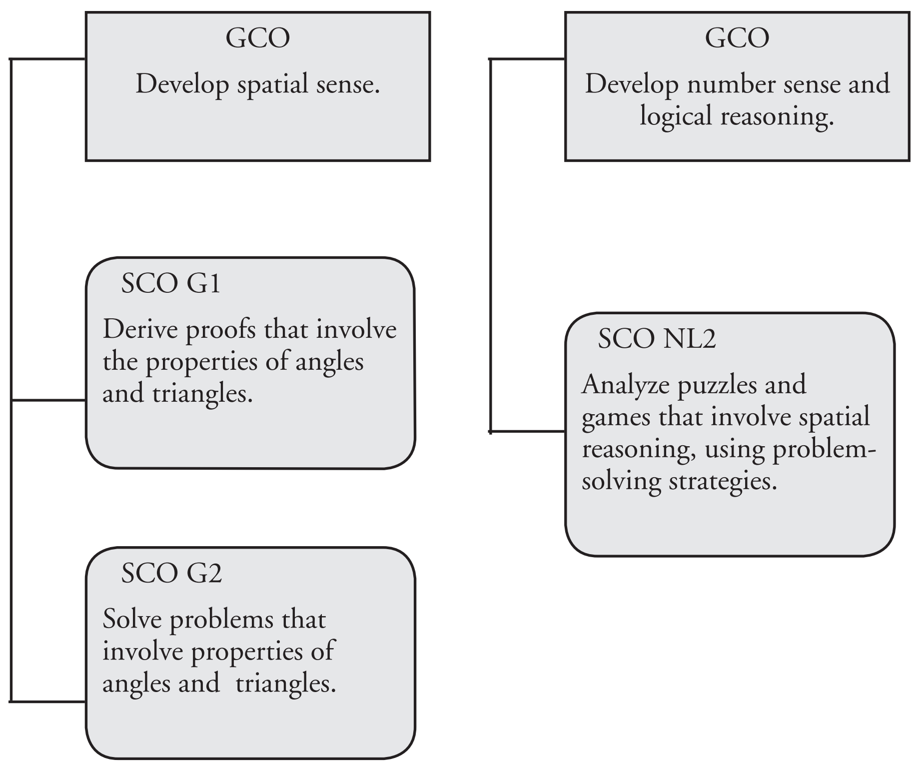
Unit Overview

Focus and Context

In this unit, students will identify relationships among the measures of angles formed by parallel lines cut by a transversal. They will also prove deductively, the properties of angles in a triangle and other polygons.

Students will explore the different ways to prove that two triangles are congruent. They will then use this information to complete geometric proofs.

Outcomes Framework



Process Standards Key

[**C**] Communication
[**CN**] Connections
[**ME**] Mental Mathematics and Estimation
[**PS**] Problem Solving
[**R**] Reasoning
[**T**] Technology
[**V**] Visualization

SCO Continuum

Mathematics 1201	Mathematics 2201	Mathematics 3201
	Geometry	
	G1 Derive proofs that involve the properties of angles and triangles. [CN, R, V] G2 Solve problems that involve properties of angles and triangles. [CN, PS, V]	
	Number and Logic	Logical Resoning
	NL2 Analyze puzzles and games that involve spatial reasoning, using problem-solving strategies. [CN, PS, R, V]	LR1 Analyze puzzles and games that involve numerical and logical reasoning, using problem-solving strategies. [CN, ME, PS, R]

Geometry

Outcomes

Students will be expected to

G1 Derive proofs that involve the properties of angles and triangles

[CN, R, V]

Achievement Indicator:

G1.1 *Generalize, using inductive reasoning, the relationships between pairs of angles formed by transversals and parallel lines, with or without technology.*

Elaborations—Strategies for Learning and Teaching

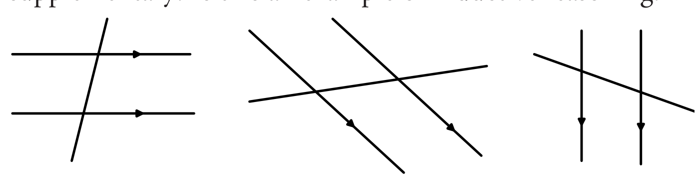
In Grade 6, students learned that the sum of the angles in a triangle is equal to 180°(6SS2). They will now use deductive reasoning to prove that this is true for all triangles. In Grade 7, students identified parallel and perpendicular lines and used various strategies to draw a line segment that was perpendicular (or parallel) to a given line segment (7SS3). In Grade 6, students were introduced to straight angles but they are not familiar with the terms complementary angles, supplementary angles or vertically opposite angles. They are familar with the term congruent through their work with congruent polygons.

Students should be given an opportunity to reflect on and discuss the following:

- Where in real-life do you see parallel lines?
- Provide an example of a situation where it is important for lines to be parallel. What would happen if they were not parallel?

Students should be introduced to the term transversal before developing the various angle properties. The following questions can be used to promote discussion:

- How many angles are formed when a transversal intersects two parallel lines?
- Ask students to measure all of the angles and note the patterns they observe. Which angles are equal? Which angles are supplementary? Is this an example of inductive reasoning?



Identify the names of the angles and then allow the students to generalize the relationships that exist when a set of parallel lines is intersected by a transversal. Ask them to state their conjectures and share their responses with the class.

- Corresponding angles are equal.
- Vertically opposite angles are equal.
- Alternate interior angles are equal.
- Alternate exterior angles are equal.
- Interior angles on the same side of the transversal are supplementary.

Even though all the relationships between angles are explored, students will initially use vertically opposite angles and corresponding angles to prove deductively the other types of angles later in this unit. Students generally have little difficulty identifying corresponding angles, therefore, this will be a starting point when they prove the other types.

General Outcome: Develop spatial sense.

Suggested Assessment Strategies

Performance

- Use tape to create two parallel lines with a transversal on the floor. Students should work in pairs. Student A stands on the angle. Student B picks a card to direct Student A to move to an adjacent angle, vertically opposite angle, etc. Students switch roles after five moves.

(G1.1)

Journal

- Ask students to find an object in their environment that models parallel lines cut by a transversal. Sketch the object and highlight the various pairs of angles that are formed.

(G1.1)

Resources/Notes

Authorized Resource

Principles of Mathematics 11

2.1: Exploring Parallel Lines

Student Book (SB): pp.70-72
Teacher Resource (TR): pp.70-75

Web Link

An interactive game where students can explore and discover the rules for angles of parallel lines cut by a transversal.
http://www.mathwarehouse.com
(Keyword: parallel line and angle)

Geometry

Outcomes

Students will be expected to

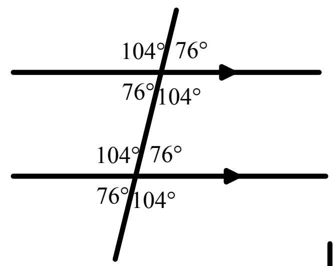
G1 Continued ...

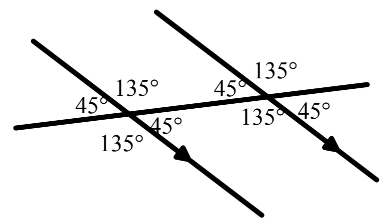
Achievement Indicator:

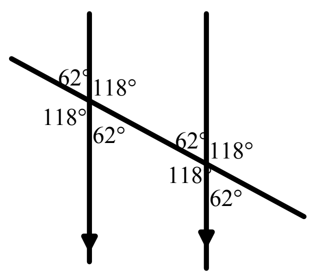
G1.1 *Continued*

Elaborations—Strategies for Learning and Teaching

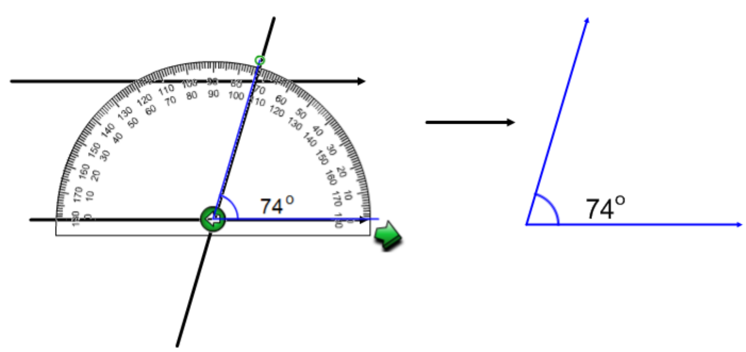
Software programs such as FX Draw and Geometer's Sketchpad could also be used to develop the angle relationships. The construction of a geometric situation is as powerful for learning as the analysis of the situation. This alternative to paper and pencil creates a dynamic learning environment where students can interactively investigate and generalize the relationship between pairs of angles formed by transversals and parallel lines.







An interactive whiteboard could also be used to develop these relationships. Students could be invited to use the interactive protractor to measure all the angles.



After exploring multiple examples, students make their generalizations about the angle relationships that exist.

General Outcome: Develop spatial sense.

Suggested Assessment Strategies

Observation

- Ask students to look for examples of parallel lines cut by a transversal around their school. Some examples might include court lines on the gymnasium floor or floor tile patterns in the classroom. Students could trace the lines onto a piece of paper and then measure the angles using a protractor to determine relationships amongst the angles.

(G1.1)

Resources/Notes

Authorized Resource

Principles of Mathematics 11

2.1: Exploring Parallel Lines

SB: pp.70-72
TR: pp.70-75

Geometry

Outcomes

Students will be expected to

G1 Continued ...

Achievement Indicators:

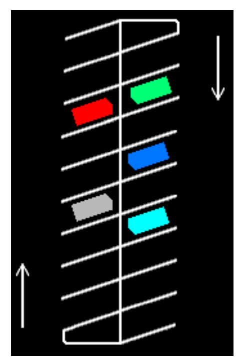
G1.1 *Continued*

Elaborations—Strategies for Learning and Teaching

As an alternative activity, students could investigate these angles using a parking lot. It would be beneficial if students were exposed to a parking lot where all the angles are not right angles. When workers paint lines for a parking lot, they aim to paint lines that are parallel to each other. The lines in a parking lot, therefore, provide an ideal illustration of the relationship between angles created by parallel lines and a transversal. Using chalk, students can discuss and mark the different types of angles, for example, in the school's parking lot. This activity could also be set up in the gymnasium using tape to represent the parking spaces.They can then measure the angles to determine which angles are equal and which are supplementary.

This would be a good opportunity to promote discussion by asking students the following questions:

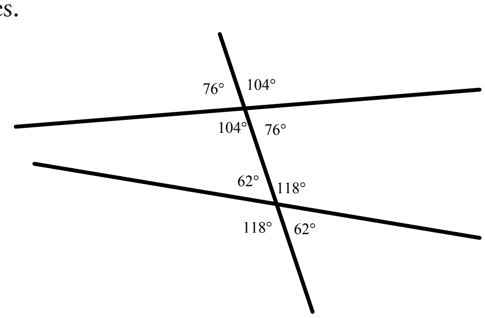
- Why would a parking lot have parallel lines that intersect at non-right angles?
- Why would a parking lot have one way traffic?



G1.2 *Verify, with examples, that if lines are not parallel, the angle properties do not apply.*

Ask students if their conjecture about angle measures holds true if a transversal intersects a pair of non-parallel lines. Encourage students to construct non-parallel lines with the aid of technology and to use the measuring tool to measure the angles.

Students should recognize when a tranversal intersects a pair of non-parallel lines, the angle properties do not apply except for vertically opposite angles.



General Outcome: Develop spatial sense.

Suggested Assessment Strategies

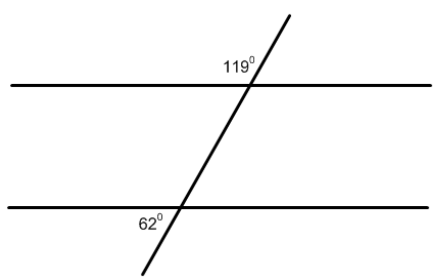
Performance

- Ask students, in groups of three or four, to draw five sets of lines cut by a transversal. Draw the first four sets with nonparallel lines crossed by a transversal, with each set getting closer to being parallel. The fifth set should be parallel lines crossed by a transversal. Number the sets from 1 to 5. Measure the same pair of alternate interior angles. Also have them find the sum of the same-side exterior angles in each set of lines and to record their data in an organized chart. After reviewing and discussing the data in their chart, students should draw conclusions about the measures of alternate interior angles and the sum of same-side exterior angles for lines that are not parallel and lines that are parallel.

(G1.2)

Paper and Pencil

- Ask students to determine if the lines shown below are parallel. They should explain their reasoning.



(G1.2)

Resources/Notes

Authorized Resource

Principles of Mathematics 11

2.1: Exploring Parallel Lines

SB: pp.70-72
TR: pp.70-75

Web Link

www.k12pl.nl.ca/seniorhigh/introduction/math2201/classroomclips.html

In the clip, **Exploring Parallel Lines,** students investigate the relationship between pairs of angles formed by parallel lines and a transveral.

Geometry

Outcomes

Students will be expected to

G2 Solve problems that involve properties of angles and triangles.

[CN, PS, V]

Achievement Indicator:

G2.1 *Construct parallel lines, given a compass or a protractor, and explain the strategy used.*

Elaborations—Strategies for Learning and Teaching

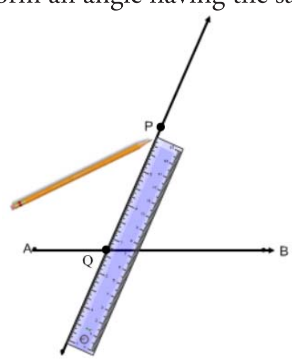
Students should have an opportunity to construct parallel lines and verify the angle relationships that were developed earlier. They can use a variety of strategies, such as a compass, protractor, or paper folding and protractor, to ensure lines are parallel. Consider the following:

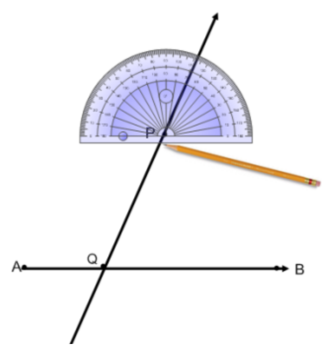
- Using a protractor

Step 1: Draw a line AB. Place a point P above the line.

Step 2: Draw a line through P and passing through AB. Label the intersection point Q.

Step 3: Using a protractor measure $\angle PQB$. Use your protractor to then form an angle having the same measure at P.





- Paper folding and a protractor.

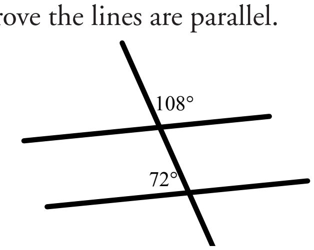
Step 1: Take a blank sheet of paper and fold it in half.

Step 2: Fold it in half again.

Step 3: Unfold the paper and trace any 2 of the fold lines, using a ruler and protractor.

Step 4: Construct a transversal.

Students should be encouraged to verify that the lines are parallel by measuring the angles they have created (corresponding, vertically opposite, alternate interior, alternate exterior, interior angles on the same side of the transversal). Ask students how many angle relationships are necessary to measure to prove the lines are parallel.

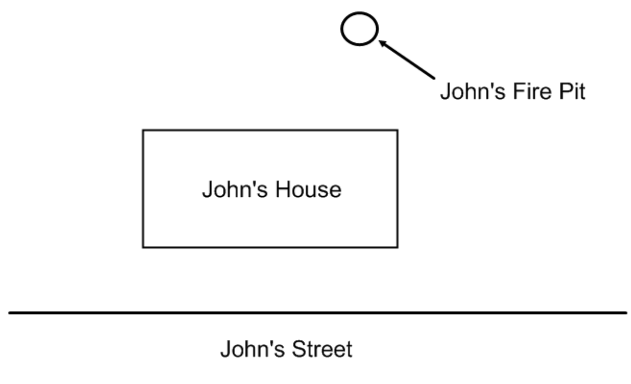


General Outcome: Develop spatial sense.

Suggested Assessment Strategies

Paper and Pencil

- John is building a fence in his backyard and plans to construct the fence parallel to his house. Using the diagram shown below, ask students where would John build the fence?



(G2.1)

Resources/Notes

Authorized Resource

Principles of Mathematics 11

2.2: Angles Formed by Parallel Lines

SB: pp.73-85
TR: pp.76-86

Geometry

Outcomes

Students will be expected to

G1 Continued...

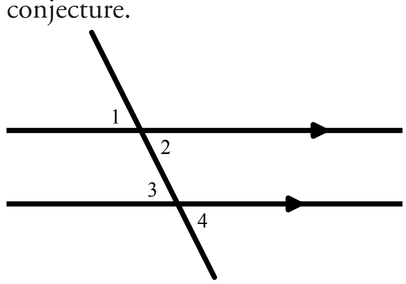
Achievement Indicators:

G1.3 *Prove, using deductive reasoning, properties of angles formed by transversals and parallel lines, including the sum of the angles in a triangle.*

G1.4 *Identify and correct errors in a given proof of a property that involves angles.*

Elaborations—Strategies for Learning and Teaching

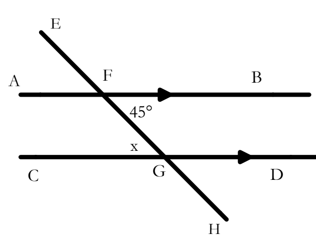
Students have explored the angle relationships when two parallel lines are cut by a tranversal. They will now use their knowledge of corresponding angles, vertically opposite angles and supplementary angles to formally prove the other relationships, such as alternate interior angles. A two-column proof and a paragraph proof are two of the most common strategies used to construct proofs involving properties of angles formed by transversals and parallel lines. Teachers could ask students to make a conjecture that involves alternate exterior angles formed by parallel lines and a transversal. Students may conjecture that alternate exterior angles are equal and use the following to prove their conjecture.



Statement	Justification
$\angle 1 = \angle 2$	vertically opposite angles
$\angle 2 = \angle 4$	corresponding angles
$\angle 1 = \angle 4$	transitive property

It is beneficial to have students analyze solutions that contain errors, explain why errors might have occurred and how they can be corrected. This reinforces the angle relationships that have been developed throughout this unit. Students should be able to identify and correct errors such as those present in the following example:

Determine the measure of *x*.



Statement	Justification
$\angle BFG = 45°$	given
$\angle BFG = \angle FGD$	interior angles on the same side of the tranversal are equal
$\angle FGD + \angle FGC = 180°$ $\angle FGC = 180° - \angle FGD$ $= 180° - 45°$ $= 135°$	supplementary angles (angles forming a straight line)

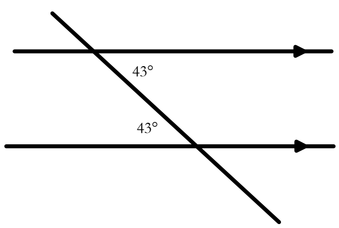
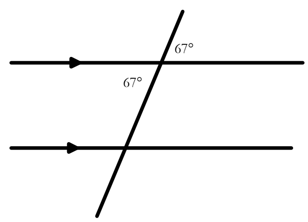
Students should recognize that interior angles on the same side of the transversal are not equal. They are supplementary angles.

General Outcome: Develop spatial sense.

Suggested Assessment Strategies

Observation

- Students can work in groups of six to play the game "I Have, Who Has". Each student is given a card such as the following:

I have....  Who has Vertically Opposite Angles?	I have.....  Who has Corresponding Angles?

Student #1 begins by identifying the pair of angles illustrated in their diagram and reading the "Who Has..." statement at the bottom of their card. All other students must then look at their pictures to see who has the card illustrating that particular pair of angles. Play continues until the game comes back to the original card.

(G1.3)

Paper and Pencil

- Ask students to create a graphic organizer highlighting the relationships between:
  - (i) Vertically opposite angles
  - (ii) Alternate interior angles
  - (iii) Alternate exterior angles
  - (iv) Corresponding angles
  - (v) Same side interior angles

(G1.3)

Resources/Notes

Authorized Resource

Principles of Mathematics 11

2.2: Angles Formed by Parallel Lines

SB: pp.73-85
TR: pp.76-86

Geometry

Outcomes

Students will be expected to

G2 Continued...

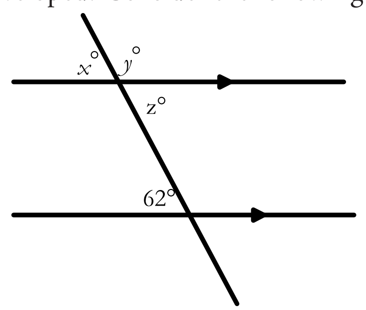
Achievement Indicators:

G2.2 *Determine the measures of angles in a diagram that includes parallel lines, angles and triangles, and justify the reasoning.*

G2.3 *Determine if lines are parallel given the measure of an angle at each intersection formed by the lines and a transversal.*

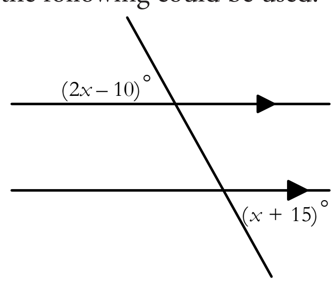
Elaborations—Strategies for Learning and Teaching

Students should be exposed to diagrams where they are asked to determine the measures of unknown angles using the relationships that have been developed. Consider the following example:



Students should also have an opportunity to work backwards and determine if lines are parallel when given angle measurements. A common student error occurs when they identify lines as being parallel based only on vertically opposite angles that are equal.

As an extension, students could explore situations where angles are expressed using variable expressions. This type of problem will require students to solve a linear equation. Students will determine the value of x and then use substitution to determine the angle measures. An example such as the following could be used.

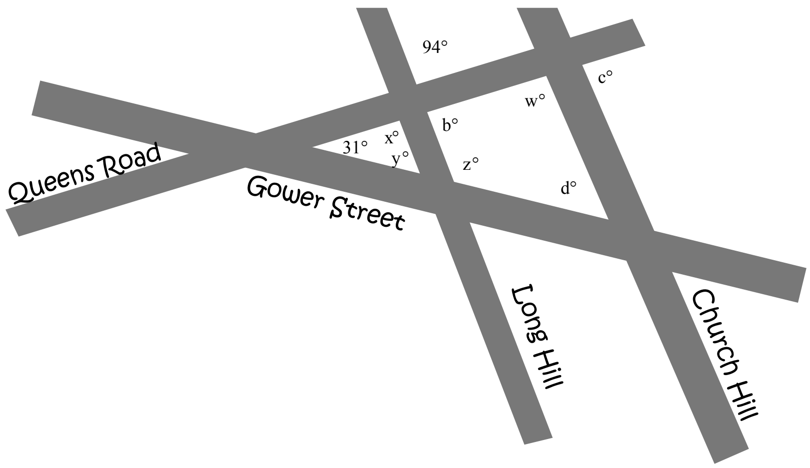


General Outcome: Develop spatial sense.

Suggested Assessment Strategies

Paper and Pencil

- Long Hill and Church Hill are parallel to each other. Ask students to determine the missing measures on the map shown below:



(G2.2)

- Ask students to design a map involving parallel lines cut by a transversal. Provide at least one angle measurement and create a question to identify unknown angles.

(G2.2)

Resources/Notes

Authorized Resource

Principles of Mathematics 11

2.1: Exploring Parallel Lines

SB: pp.70-72
TR: pp.70-75

2.2: Angles Formed By Parallel Lines

SB: pp.73-85
TR: pp.76-86

Geometry

Outcomes

Students will be expected to

G2 Continued...

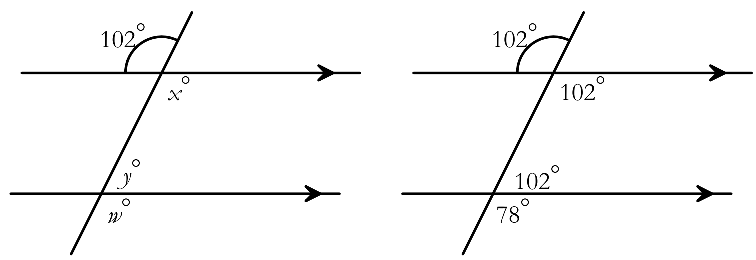
Achievement Indicators:

G2.4 *Identify and correct errors in a given solution to a problem that involves the measures of angles.*

G2.5 *Solve a contextual problem that involves angles or triangles.*

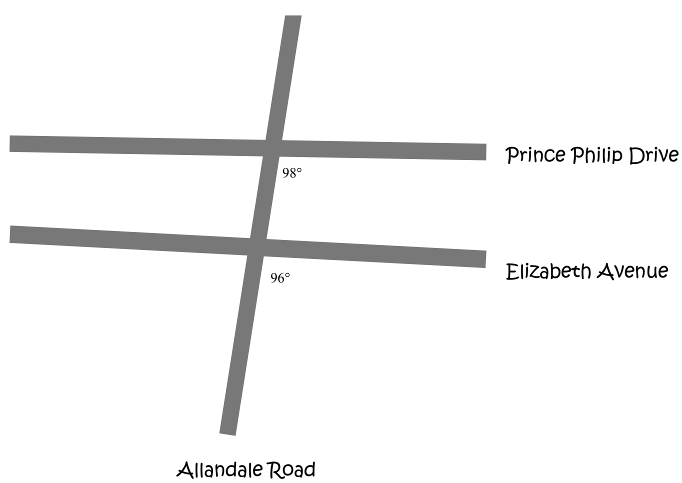
Elaborations—Strategies for Learning and Teaching

A common error occurs when students incorrectly identify pairs of angles, leading to an incorrect measurement. For example, students identify same side interior angles as equal rather than supplementary. Students should be able to identify and correct errors such as those present in the following example:



Students should be exposed to problems where they can make a connection between mathematics and their environment. Consider the following example:

Suppose Prince Philip Drive and Elizabeth Avenue follow a straight line path and intersect Allandale Road at angles of 98° and 96° as shown in the map below. If the streets were to continue in a straight line, would their paths ever cross? Explain your reasoning.



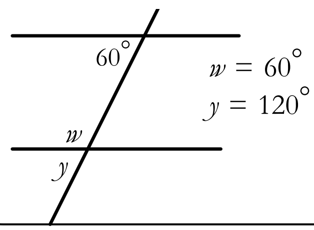
General Outcome: Develop spatial sense.

Suggested Assessment Strategies

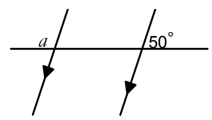
Performance

- Quiz-Quiz-Trade: Each student is given a card with a problem. The answer is written on the back of the card. In groups of two, partner A asks the question and partner B answers. They switch roles and repeat. Partners trade the cards and then find a new partner. Sample cards are shown below.

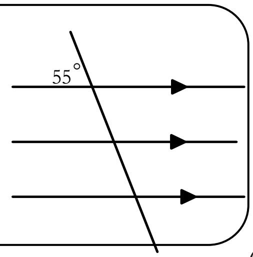
Identify and correct the error.



What is the value of *a*?



How many angles in the diagram measure 55° ?



(G2.2, G2.4)

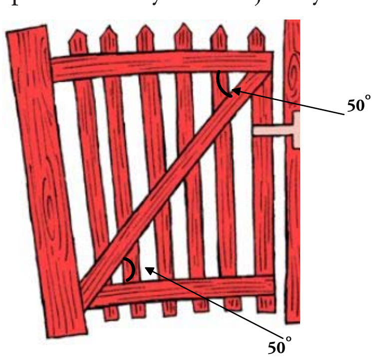
Observation

- Create centers in the classroom containing solutions to problems that involve the measures of angles. Students will participate in a carousel activity where they are asked to move throughout the centers to identify and correct the errors.

(G2.4)

Paper and Pencil

- Ask students whether the given angle measure proves that the lines are parallel. They should justify their answer.



(G2.4, G2.5)

Resources/Notes

Authorized Resource

Principles of Mathematics 11

2.1: Exploring Parallel Lines

SB: pp.70-72
TR: pp.70-75

2.2: Angles Formed By Parallel Lines

SB: pp.73-85
TR: pp.76-86

Web Link

www.k12pl.nl.ca/seniorhigh/introduction/math2201/classroomclips.html

In the clip, **Exploring Parallel Lines,** students participate in the activity *Quiz-Quiz-Trade* related to missing measures, error analysis and identifying pairs of angles when parallel lines are cut by a transversal.

Geometry

Outcomes

Students will be expected to

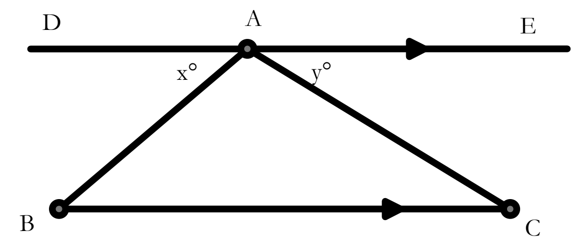
G1, G2 Continued ...

Achievement Indicators:

G1.3 *Continued*

Elaborations—Strategies for Learning and Teaching

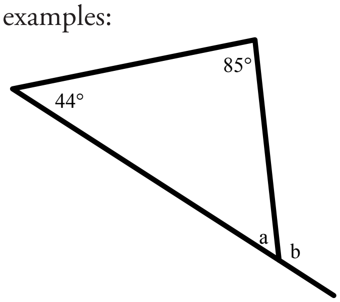
Students will prove, using deductive reasoning, that the sum of the interior angles of any triangle is 180°. Give students direction by asking them to draw a triangle and to draw a line which is parallel to one of the sides of the triangle and tangent at one of the vertices.

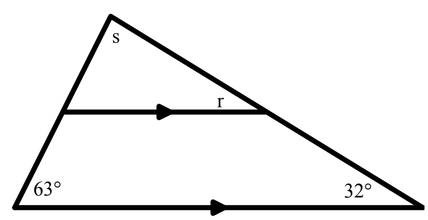


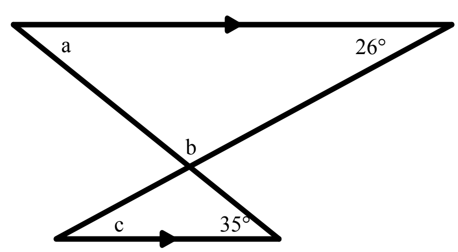
Students should be able to complete the proof using the properties of angles formed by transversals and parallel lines.

G2.2 *Continued*

Earlier in this section, students were exposed to diagrams where they were asked to determine missing angle measures formed by transversals and parallel lines. Students will continue to find the measures of unknown angles but now within triangles. Consider the following examples:







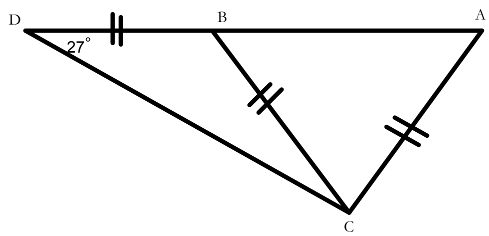
Students could be exposed to examples where variables represent the angles requiring them to solve a linear equation. In ΔABC, for example, $\angle A = 6x - 30$, $\angle B = -x + 50$, and $\angle C = 3x$. Ask students to determine the measure of $\angle A$.

General Outcome: Develop spatial sense.

Suggested Assessment Strategies

Paper and Pencil

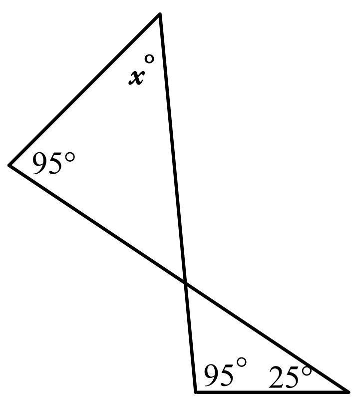
- Ask students to determine the value of ∠A and explain their reasoning.



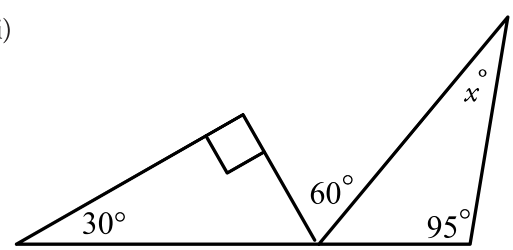
(G1.3, G2.2)

- Ask students to determine the missing measures:

(i)



(ii)



(G1.3)

Journal

- Mckenzie said that if a triangle is obtuse, two of the angles of the triangle are acute. Ask students if they agree with Mckenzie. They should explain their reasoning.

(G1.3)

Resources/Notes

Authorized Resource

Principles of Mathematics 11

2.3: Angles Properties in Triangles

SB: pp.86-93
TR: pp.87-93

Geometry

Outcomes

Students will be expected to

G1 Continued...

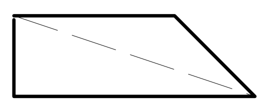
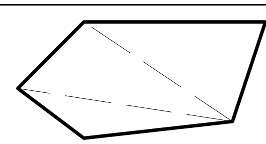
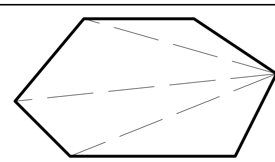
Achievement Indicator:

G1.5 *Generalize, using inductive reasoning, a rule for the relationship between the sum of the interior angles and the number of sides (n) in a polygon, with or without technology.*

Elaborations—Strategies for Learning and Teaching

Using a visual representation, students should distinguish between convex and non-convex (concave) polygons. The focus of this chapter, however, will be on convex polygons.

Encourage students to discover the relationship between the sum of the interior angles and the number of sides in a convex polygon using the angle sum property. They are aware that the sum of the angles in a triangle is 180°. Students can separate each polygon into triangles by drawing diagonals and use the following table to help them with their investigation. Each vertex of a triangle must be a vertex of the original polygon.

Number of Sides	Diagram	Number of Triangles Formed	Sum of Angles
4		2	360°
5		3	540°
6		4	720°

The focus of this investigation is for students to recognize that the sum of the angles increases by 180° as the number of sides increase by one. They should also observe that the number of triangles formed is always two less than the number of sides in the polygon. Using this information, encourage students to develop a formula for the sum of the measures of the interior angles of a polygon, $S = 180°(n - 2)$ where S represents the sum of the interior angles and n is the number of sides of the polygon.

If the polygon is regular, students can use their knowledge of the sum of the measures of the angles in a polygon to determine the measure of each interior angle.

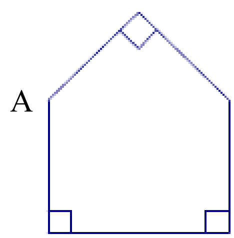
General Outcome: Develop spatial sense.

Suggested Assessment Strategies

Paper and Pencil

- Ask students to answer the following:

  (i) In baseball, the home plate is shaped like the one shown. It has 3 right angles and 2 other congruent angles (A and B). Find the measures of $\angle A$ and $\angle B$.



(G1.5)

  (ii) Ask students, using at least three examples, to verify inductively that the conjectures regarding the sum of interior angles of a polygon are valid for convex polygons.

(G1.5)

Resources/Notes

Authorized Resource

Principles of Mathematics 11

2.4: Angle Properties in Polygons

SB: pp.94-103
TR: pp.94-104

Geometry

Outcomes

Students will be expected to

G1 Continued ...

Achievement Indicator:

G1.6 *Prove, using deductive reasoning, that two triangles are congruent.*

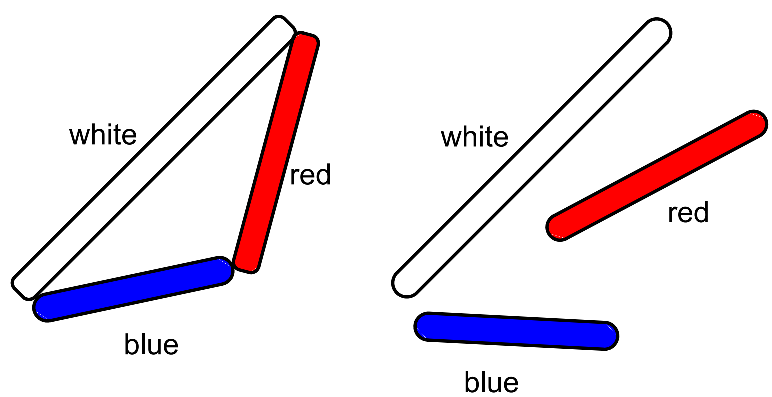
Elaborations—Strategies for Learning and Teaching

In Grade 9, students were exposed to similar triangles. They have also been exposed to the concept of congruent polygons (9SS3). However, the term congruent and how it relates directly to congruent triangles should be revisited with students.

If two triangles are congruent, their corresponding parts are congruent. Encourage students to investigate the minimum amount of information needed to prove that two triangles are congruent. The following activity will help them conceptualize congruent triangles.

- Provide students with six pieces of pipe cleaners to represent the sides of the triangle and three foam wedges (or construction paper) to represent the angles. The pipe cleaners should be colour-coordinated using various sizes (2 long white pieces, 2 medium red pieces, 2 small blue pieces) to help students identify congruent parts.
- Ask students to construct a triangle where each side is a different colour pipe cleaner. This will be the original triangle.
- The first exploration will involve students trying to construct another triangle with the same side lengths as the original.

Ask students if their construction is identical to the original and did they need to take the orientation of the triangle into consideration.



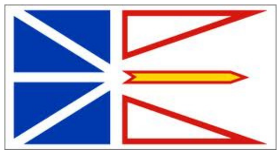
Students should notice that no matter where they put the sides, a congruent triangle is constructed. This will lead into a discussion around the side-side-side relationship (SSS) of congruent triangles.

General Outcome: Develop spatial sense.

Suggested Assessment Strategies

Paper and Pencil

- Ask students to answer the following questions:

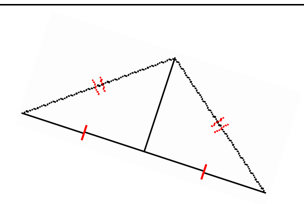
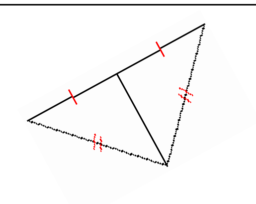
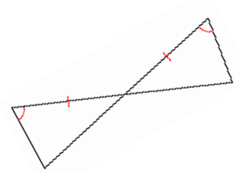
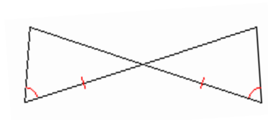
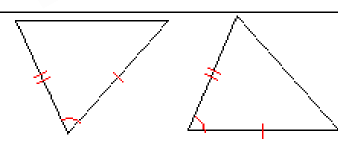
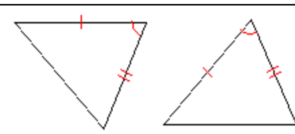
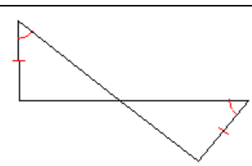
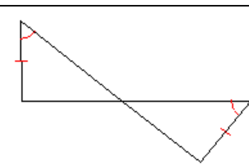


(i) How many triangles are used in the NL flag?
(ii) Identify any pairs of congruent triangles. Prove they are congruent.

(G1.6)

Performance

- Ask students to participate in the Memory game where they have to find three matching cards. One card will state a postulate for congruent triangles (SSS, SAS, etc.) and the other two cards show sample triangles which satisfy the postulate. In groups of two, students will take turns trying to find the three matching cards for each postulate and the student at the end of the game with the most cards wins.

SSS		
ASA		
SAS		
AAS		

(G1.6)

Resources/Notes

Authorized Resource

Principles of Mathematics 11

2.5: Exploring Congruent Triangles

SB: pp.104-106
TR: pp.105-110

2.6: Proving Congruent Triangles

SB: pp.107-123
TR: pp.111-136

Web Links

www.k12pl.nl.ca/seniorhigh/introduction/math2201/classroomclips.html

In the **clip, Exploring Congruent Triangles (Part 1)**, students investigate the infomation needed to prove two triangles are congruent.

This interactive game allows students to explore the conditions to create congruent triangles.
http://illuminations.nctm.org
(Keyword: congruent theorems)

Geometry

Outcomes

Students will be expected to

G1 Continued ...

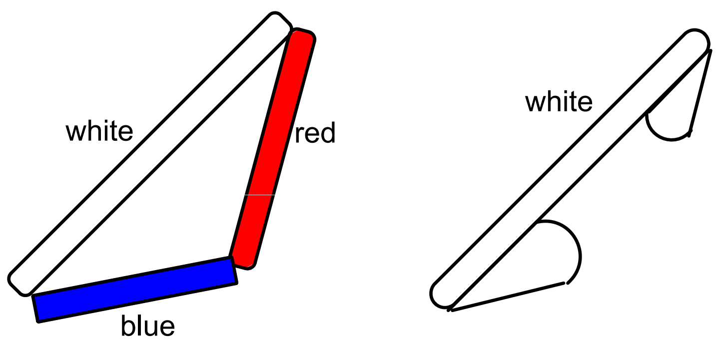
Achievement Indicator:

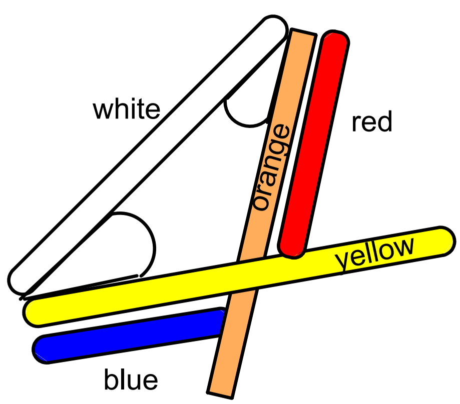
G1.6 *Continued*

Elaborations—Strategies for Learning and Teaching

To continue with this investigative approach, students should explore other relationships, such as angle-side-angle (ASA). Consider the following activity:

- The materials are the same in the previous activity, but they will need additional pipe cleaners (yellow and orange). Students should not assume the red and blue pipe cleaner will be required to complete the triangle.
- Ask students to use a pipe cleaner and attach a wedge to each end. They will use the extra pipe cleaners to construct the other two sides of the triangle.
- Students will compare the side lengths of their construction to the side lengths of the original using the red and blue pieces of the pipe cleaner.





Prompt discussion using questions such as: Are the triangles congruent? What did you notice about the side in relation to the the two angles?

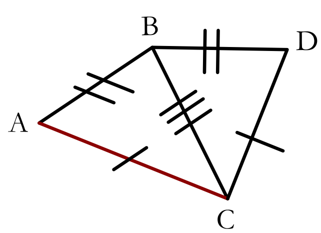
General Outcome: Develop spatial sense.

Suggested Assessment Strategies

Performance

- Ask students to participate in the following activity, called *Tile Design*. Your task is to design a tile approximately 8 cm by 8 cm. The drawings on the tile must meet the following specifications:
  - two congruent obtuse triangles which demonstrate congruency by ASA.
  - two congruent scalene triangles which demonstrate congruency by SSS.
  - two congruent isosceles right triangles which demonstrate congruency by SAS.
  - two congruent acute triangles which demonstrate congruency by AAS.
  - one equilateral triangle.

  The finished tile should have exactly nine connected triangles in its design (i.e., the two acute triangles cannot double as your two scalene triangles). Students should label and mark each pair of triangles as shown below.



$\overline{AB} \cong \overline{DB}$

$\overline{AC} \cong \overline{DC}$

$\overline{BC} \cong \overline{BC}$

$\Delta ABC \cong \Delta DBC$

The two scalene triangles are congruent by SSS.

(G1.6)

Resources/Notes

Authorized Resource

Principles of Mathematics 11

2.5: Exploring Congruent Triangles

SB: pp.104-106
TR: pp.105-110

2.6: Proving Congruent Triangles

SB: pp.107-123
TR: pp.111-136

Geometry

Outcomes

Students will be expected to
G1 Continued ...

Achievement Indicator:

G1.6 *Continued*

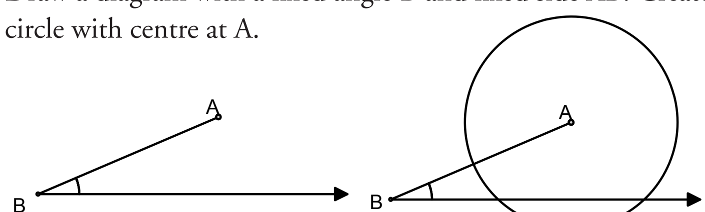
Elaborations—Strategies for Learning and Teaching

It is important for students to continue this investigation as they explore the side-angle-side relationship (SAS) and the side-angle-angle relationship (SAA). Use questions to help students work through the activity: Can congruent triangles be constructed using these properties? What three pieces of information are required to prove congruent triangles?

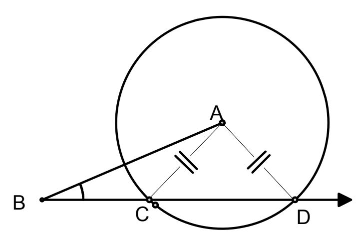
Using the investigative approach, students should have recognized that there are four shortcuts that can be used to prove congruent triangles: SSS, SAS, ASA and SAA. If the lengths of three sides of a triangle are given, for example, only one unique triangle can be constructed. As a result, any triangles constructed with these side lengths will be congruent (SSS property).

Students should also be given time to investigate other shortcuts. Students may determine the SSA property only works for a right triangle, known as the Hypotenuse-Leg Theorem. Although this theorem is not part of this outcome, discussion may be warranted here as to why it works for a right triangle.

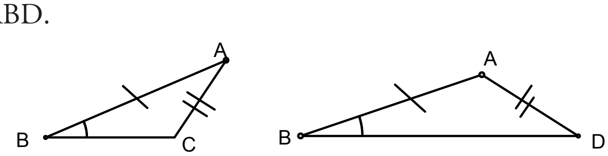
- Draw a diagram with a fixed angle B and fixed side AB. Create a circle with centre at A.



- Make a triangle by drawing two lines that are radii of the circle.



- Two triangles are constructed. However, ΔABC is not congruent to ΔABD.



Using this construction, students should recognize that knowing only side-side-angle (SSA) does not work because the unknown side could be located in two different places.

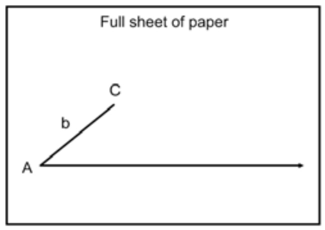
General Outcome: Develop spatial sense.

Suggested Assessment Strategies

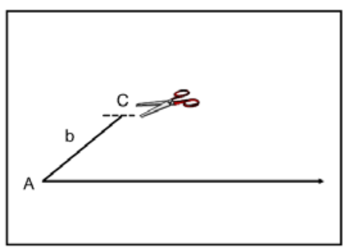
Performance/Observation

- Ask students to complete the following activity.

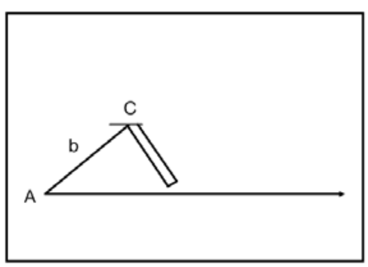
Step 1: Provide or have students construct a diagram similar to the one below, where side *b* and angle A are fixed.



Step 2: Cut a slit in the paper at point C large enough to fit a small strip of paper. This strip will represent side "*a*" of the triangle.



Step 3: Insert the strip into the slot at C as shown.



Step 4: Have students explore various lengths for side "a" by pulling the strip out from C and rotating it left and right.

As teachers observe, consider the following questions:

(i) How many triangles can be formed when the strip is too short? (no triangle)

(ii) How many triangles can be formed when the strip is perpendicular to side c? (one right triangle)

(iii) Create a triangle using a longer strip. Can another triangle be created using the same strip length? (two unique triangles)

(G1.6)

Resources/Notes

Authorized Resource

Principles of Mathematics 11

2.5: Exploring Congruent Triangles

SB: pp.104-106
TR: pp.105-110

2.6: Proving Congruent Triangles

SB: pp.107-123
TR: pp.111-136

Web Link

www.k12pl.nl.ca/seniorhigh/introduction/math2201/classroomclips.html

In the **clip, Exploring Congruent Triangles (Part 2)**, students investigate the SSA relationship.

Geometry

Outcomes

Students will be expected to

G1 Continued ...

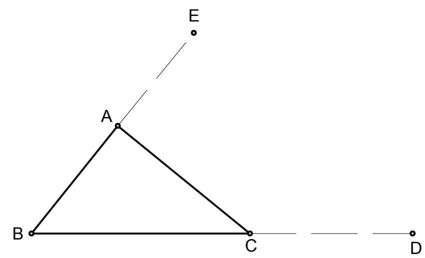
Achievement Indicator:

G1.6 *Continued*

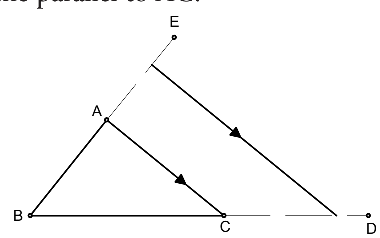
Elaborations—Strategies for Learning and Teaching

Next consider the AAA case.

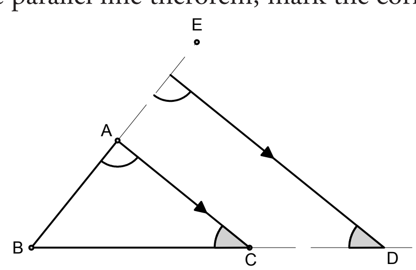
- Draw a triangle ABC. Extend the lines on two sides of the triangle.



- Draw a line parallel to AC.



- Using the parallel line therorem, mark the corresponding angles.



- Two triangles are constucted. However, ΔABC is clearly not congruent to ΔEBD.

From this, students should recognize that knowing only angle-angle-angle can produce similar triangles but not congruent triangles.

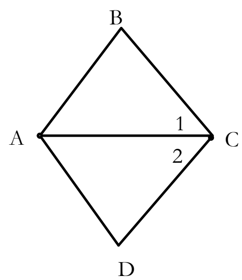
Students will focus on formal geometric proofs. Where applicable, it would be beneficial to draw diagrams so that students can have a visual representation of what is given and can determine other relationships. Remind students that, when they prove two triangles are congruent, their corresponding parts are congruent. Students should be exposed to examples with triangles that have a common side.

General Outcome: Develop spatial sense.

Suggested Assessment Strategies

Paper and Pencil

- Ask students to answer the following:



Given $BC = CD$

$\overline{AC}$ bisects $\angle BCD$

Prove $\Delta ABC \cong \Delta ADC$

(G1.6)

Performance

- This activity will require about ten sets of geometric proofs that have been written, cut apart, and put in separate envelopes. Place students in groups of two or three.

  (i) Teachers distribute a diagram and something to prove to each group to complete. They should verify the proof is logical and has no errors.
  (ii) Ask students to cut apart their proof, separating their statements and reasons. Place the pieces, along with its diagram, in an envelope.
  (iii) Each group will switch envelopes with another group in their class.
  (iv) Ask students to put the proof puzzles together in the correct order by organizing the statements with the correct reason.
  (v) A recorder in the group will write the finished product for the group to submit to the teacher.
  (vi) When a group finishes one proof, they put the puzzle back in the envelope, exchange with another group and repeat the process. Each group should be able to complete three or four proofs in a class period.

(G1.6)

Resources/Notes

Authorized Resource
Principles of Mathematics 11

2.5: Exploring Congruent Triangles

SB: pp.104-106
TR: pp.105-110

2.6: Proving Congruent Triangles

SB: pp.107-123
TR: pp.111-136

Number and Logic

Outcomes

Students will be expected to

NL2 Analyze puzzles and games that involve spatial reasoning, using problem-solving strategies. [CN, PS, R, V]

Achievement Indicators:

NL2.1 *Determine, explain and verify a strategy to solve a puzzle or to win a game; e.g.,*

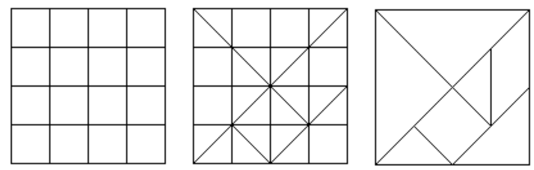
- *guess and check*
- *look for a pattern*
- *make a systematic list*
- *draw or model*
- *eliminate possibilities*
- *simplify the original problem*
- *work backward*
- *develop alternative approaches.*

NL2.2 *Create a variation on a puzzle or a game, and describe a strategy for solving the puzzle or winning the game.*

Elaborations—Strategies for Learning and Teaching

This is an outcome that students will continuously be exposed to throughout the course.

A tangram consists of seven geometrical pieces, which can be obtained by cutting a square.

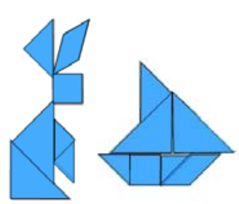


This particular tangram puzzle set is composed of seven polygons: a square, a parallelogram, and five triangles. Part of the tangram's popularity comes from its union of geometry and art. The puzzle pieces which fit together to form a square can be rearranged to form countless shapes and figures.

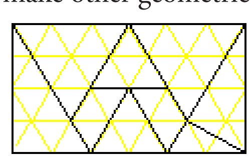
Provide students with a master so they can cut out the pieces or they could use an interactive online version or an app. Challenge students to play the tangram game to improve their thinking and geometry skills. Use the following questions to help guide students with what they should be looking for:

- Are there any distinctive features of the puzzle?
- How many different ways are there to fill in a specific piece of the puzzle? For example, a square area of the tangram could be formed using two congruent triangles.
- What happens if the parallelogram is rotated? Is the result different if you flip it?

As an alternative, ask students to work backwards to create their own unique shape using the tangram pieces. Remind them they must use seven pieces to form a shape. Encourage students to display their art work around the classroom using an appropriate title that describes the object or pattern created. Some illustrations are shown.



Students can also use tangram pieces to make other geometric figures such as a rectangle. The following is an example of a nine piece tangram.



General Outcome: Develop number sense and logical reasoning.

Suggested Assessment Strategies

Journal

- Ask students to write about their thought processes when working with the tangram puzzle. It may be helpful to prompt students using the following questions:

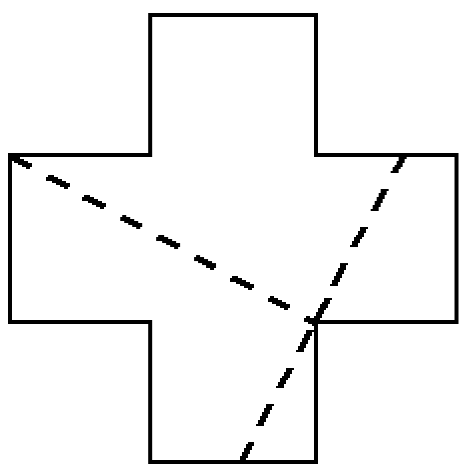
  (i) Do you have any advice for someone who was not able to assemble their shape with the puzzle pieces?

  (ii) Did you have to use any slides, flips or rotations to assemble your shape?

(NL2.1, NL2.2)

Performance

- Ask students to cut out this diagram and then cut along the dotted lines to create four pieces. Arrange the pieces to form a square.



(NL2.1, NL2.2)

- Ask students to use the traditional seven piece tangram or any other variation such as the nine piece tangram to create a unique shape. They should explain how they created their shapes and which tangram puzzle they used.

(NL2.1, NL2.2)

Resources/Notes

Authorized Resource

Principles of Mathematics 11

2.2: Angles Formed by Parallel Lines
SB: pp.73-85
TR: pp.76-86

Web Link

The interactive tangram game can be played at the following sites:

www.mathplayground.com/tangrams.html

http://nlvm.usu.edu
Keyword: tangram (9-12)

The tangram game can be played on an iPod using the following Apps: *Lets Tans*

Acute Triangle Trigonometry

Suggested Time: 9 Hours

Unit Overview

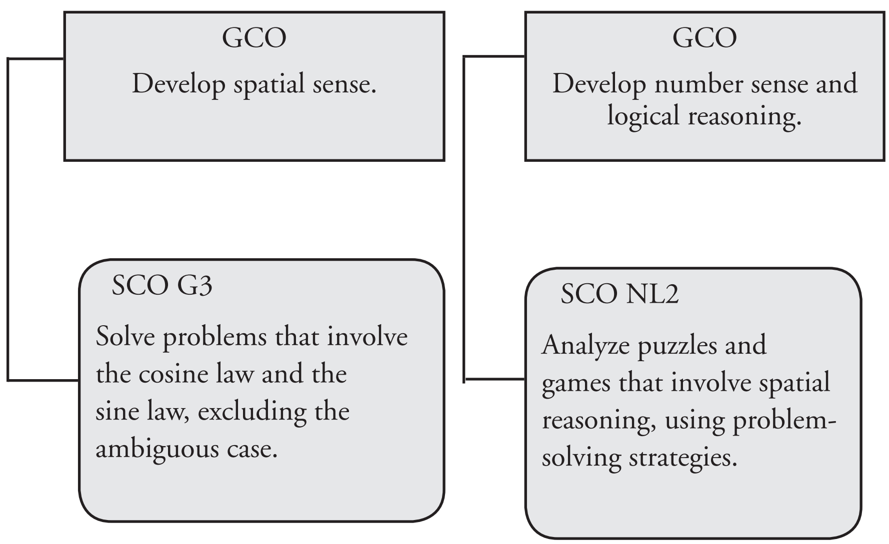
Focus and Context

Students used the primary trigonometric ratios to solve problems involving right triangles. They will now use the sine law and cosine law to solve problems that can be modelled with acute triangles, excluding the ambiguous case.

They will apply the sine law and/or cosine law to determine unknown side lengths and angle measures in acute triangles and explain their thinking about which law to use.

Students will then solve problems represented by more than one triangle. They will use a combination of strategies such as the primary trigonometric ratios, the Pythagorean theorem, the sum of the angles in a triangle, the sine law and the cosine law to solve the problem.

Outcomes Framework



Process Standards Key

[**C**] Communication	[**PS**] Problem Solving
[**CN**] Connections	[**R**] Reasoning
[**ME**] Mental Mathematics and Estimation	[**T**] Technology
	[**V**] Visualization

SCO Continuum

Mathematics 1201	Mathematics 2201	Mathematics 3201
Measurement	Geometry	
M4 Develop and apply the primary trigonometric ratios (sine, cosine, tangent) to solve problems that involve right triangles. [C, CN, PS, R, T, V]	G3 Solve problems that involve the cosine law and the sine law, excluding the ambiguous case. [CN, PS, R]	
	Number and Logic	Logical Reasoning
	NL2 Analyze puzzles and games that involve spatial reasoning, using problem-solving strategies. [CN, PS, R, V]	LR1 Analyze puzzles and games that involve numerical and logical reasoning, using problem-solving strategies. [CN, ME, PS, R]

Geometry

Outcomes

Students will be expected to

G3 Solve problems that involve the cosine law and the sine law, excluding the ambiguous case.

[CN, PS, R]

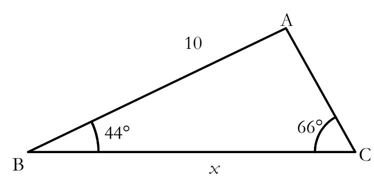
Achievement Indicator:

G3.1 *Explain the steps in a given proof of the sine law or cosine law.*

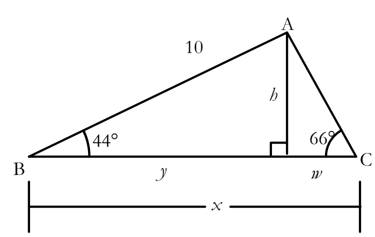
Elaborations—Strategies for Learning and Teaching

In Mathematics 1201, students used the three primary trigonometric ratios (sine, cosine and tangent) to determine the side lengths and angle measures in right triangles. In this unit, students will derive the law of sines and the law of cosines and utilize them in a number of problem-solving situations.

Students have been exposed to right-triangle trigonometry to solve problems involving right triangles. Before introducing the sine law, it may be beneficical for students to solve, using the primary trigonometric ratios, a triangle that is not a right triangle. Encourage students to draw a diagram to represent the problem to help them gain a visual understanding of the problem. Consider the following example:



Ask students if this triangle can be divided into two right triangles and what strategies can be applied to find the indicated length. They should recognize that this requires a multi-step solution. A strategy must be developed before a solution is attempted. Drawing an altitude from vertex A, students can use the primary trigonometric ratios and the Pythagorean theorem to solve for the unknown value.



$\cos 44° = \frac{y}{10} \; (y = 7.2)$

$10^2 = h^2 + 7.2^2 \; (h = 6.9)$

$\tan 66° = \frac{6.9}{w} \; (w = 3.1)$

$x = y + w$

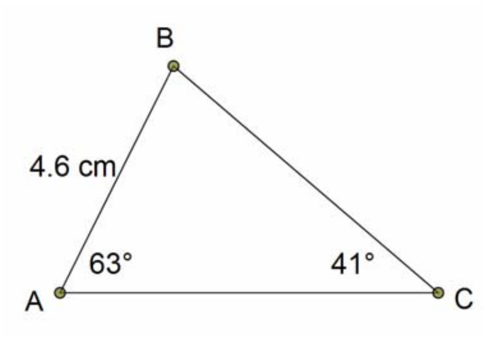
Students will be introduced in this unit to methods that are more efficient when solving an acute triangle, namely the sine law and the cosine law.

General Outcome: Develop spatial sense.

Suggested Assessment Strategies

Paper and Pencil

- Ask students to determine the length of AC in the following diagram using two different methods.



(G3.1)

Resources/Notes

Authorized Resource

Principles of Mathematics 11

3.1: Exploring Side-Angle Relationships in Acute Triangles

Student Book(SB): pp.130-131
Teacher Resource(TR): pp.143-147

3.2 Proving and Applying the Sine Law

Student Book(SB): pp.132-143
Teacher Resource(TR): pp.148-154

Geometry

Outcomes

Students will be expected to

G3 Continued...

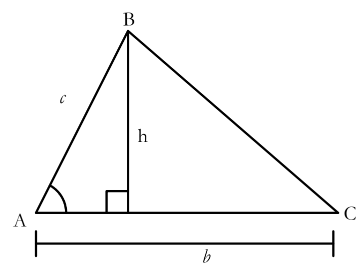
Achievement Indicator:

G3.1 *Continued*

Elaborations—Strategies for Learning and Teaching

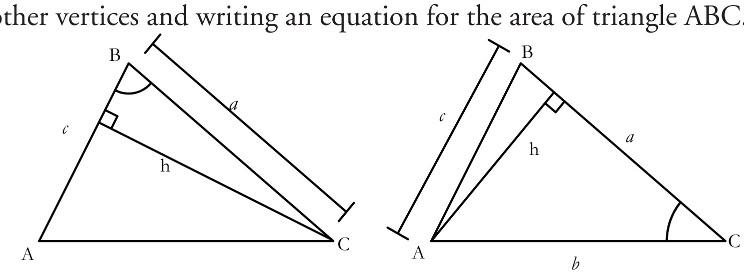
Students will prove the sine law and then use the law to solve triangles.

The Law of Sines can be derived using the area formula of a right triangle. Consider the following diagram:



Triangle ABC is not a right triangle. Therefore, students will draw an altitude from vertex B. The area formula for a right triangle is $\text{Area} = \frac{1}{2}(\text{base})(\text{height})$. Ask students to write an expression for the height (h) using the sine ratio. Since $\sin A = \frac{h}{c}$, then h = c(sinA). Therefore, $\text{Area} = \frac{1}{2}bc\sin A$.

Ask students to repeat this procedure by drawing an altitude from the other vertices and writing an equation for the area of triangle ABC.



They should conclude that $\text{Area} = \frac{1}{2}ac\sin B$ and $\text{Area} = \frac{1}{2}ab\sin C$. Promote student discussion as to why they can set the three area expressions equal to each other. Since $\frac{1}{2}bc\sin A = \frac{1}{2}ac\sin B = \frac{1}{2}ab\sin C$, students can divide each expression by $\frac{1}{2}abc$ to obtain the sine law.

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

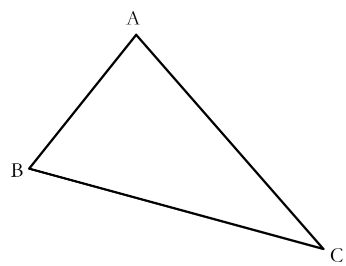
In other words, the sine law is a proportion that compares the ratio of each side of a triangle to its included angle.

General Outcome: Develop spatial sense.

Suggested Assessment Strategies

Paper and Pencil

- Provide students with a triangle and have them measure the side lengths and angles using a ruler and protractor. Students can then use inductive reasoning to prove the Law of s=Sines.



	Measure		Measure		Calculate
∠A		side a			$\frac{\sin A}{a}$
∠B		side b			$\frac{\sin B}{b}$
∠C		side c			$\frac{\sin C}{c}$

Ask students to answer the following questions:

(i) What conjecture can you make regarding the ratios calculated above?

(ii) Would your conjecture be valid if you were to use the reciprocal of the ratios?

(G3.1)

Interview

- Ask students the following questions related to the sine law:
  (i) Why does the sine law have three parts to its equation?
  (ii) Do you use all three parts at once? How can you tell which parts to use?
  (iii) How many pieces of information about a triangles side lengths and angles are needed in order to solve it using the sine law? What possible combinations will work?
  (iv) Would you use the sine law to solve right triangles? Explain.

(G3.1)

Resources/Notes

Authorized Resource

Principles of Mathematics 11

3.1: Exploring Side-Angle Relationships in Acute Triangles

SB: pp.130-131
TR: pp.143-147

3.2 Proving and Applying the Sine Law

SB: pp.132-143
TR: pp.148-154

Geometry

Outcomes

Students will be expected to

G3 Continued...

Achievement Indicators:

G3.2 *Draw a diagram to represent a problem that involves the sine law or the cosine law.*

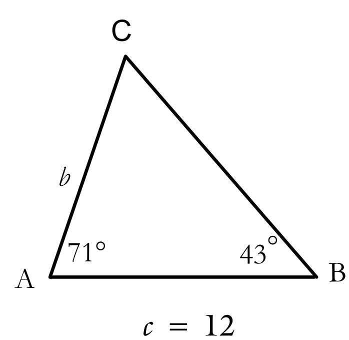
G3.3 *Solve a contextual problem that requires the use of the sine law or cosine law, and explain the reasoning.*

Elaborations—Strategies for Learning and Teaching

Students will apply the sine law to determine unknown lengths and angle measures in acute and right triangles. Encourage them to draw diagrams with both the given and unknown information marked. When working with the sine law, students will not be expected to cover the ambigous case. This occurs when you know two side lengths and a non-included angle.

Ask students what information is needed to solve problems using the sine law. The Law of Sines involves a ratio of the sine of an angle to the length of its opposite side. Students should realize that it will not work if no angle of the triangle is known or if one angle and its opposite side is not given.

When working with the sine law, students sometimes incorrectly identify side and opposite angle pairs. To avoid this error, encourage them to use arrows on the diagram when identifying the angle and its opposite side. They could also encounter problems when they cross multiply the ratios. When solving $\frac{\sin A}{12} = \frac{\sin 30°}{4}$, for example, students may incorrectly write 4sin A = sin 360°. Teachers should emphasize the use of brackets and write 4(sin A)= 12(sin 30°). Another common student error occurs when students try to solve a triangle given two angles and an included side and mistakenly think there is not enough information to use the sine law. Consider an example such as the following:



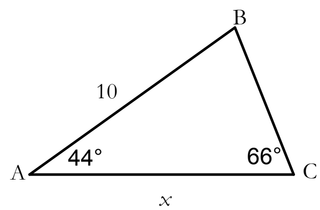
Students can use the property that the sum of the angles in a triangle is 180°. The measure of ∠C, therefore, is 66°. They can then proceed to use the sine law to find the length of side AC. Encourage students to check the reasonableness of their answer. For example, since ∠C is a little smaller than ∠A, we expect the length of side AB to be a little shorter than the length of side CB. Students should also consider asking questions such as: Is the shortest side opposite the smallest angle? Is the longest side opposite the largest angle?

General Outcome: Develop spatial sense.

Suggested Assessment Strategies

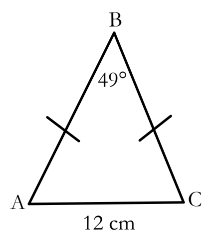
Paper and Pencil

- Ask students to answer the following:
  - (i) Two observers sight a plane at angles of elevation of 44°and 66°. If one observer is 10 km away from the plane, how far apart are the two observers from one another?



(G3.2, G3.3)

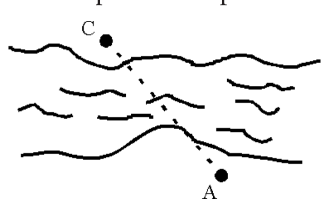
  - (ii) Find the missing side lengths in the isosceles triangle below.



(G3.2, G3.3)

Interview

- A surveyor is located on one side of a river that is impossible to cross and only has a 100 m measuring tape and a sextant (used to measure angles) in his possession. Ask students to explain how the surveyor could use only these two tools and the Law of Sines to find the distance from point A to point C.



(G3.3)

Observation

- Provide students with several practice problems using the sine law. As teachers observe students working through the problems, ask them the following questions:
  - (i) What is the unknown? Is it an angle or a side?
  - (ii) How can you isolate the unknown?
  - (iii) How can you complete the calculations?
  - (iv) How do you know whether to determine the sine or the inverse of the sine?
  - (v) Does your conclusion answer the question asked?

(G3.2, G3.3)

Resources/Notes

Authorized Resource

Principles of Mathematics 11

3.1: Exploring Side-Angle Relationships in Acute Triangles

SB: pp.130-131
TR: pp.143-147

3.2 Proving and Applying the Sine Law

SB: pp.132-143
TR: pp.148-154

Geometry

Outcomes

Students will be expected to

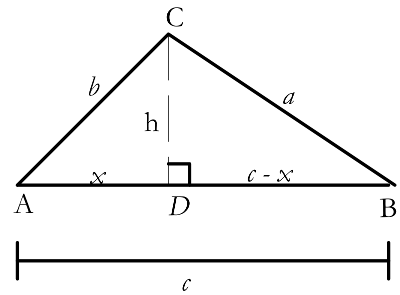
G3 Continued...

Achievement Indicator:

G3.1 *Continued*

Elaborations—Strategies for Learning and Teaching

Students will prove and use the cosine law to solve triangles. Provide students with a triangle ABC with side lengths *a*, *b* and *c* . Ask them to draw an altitude, h, from vertex C and let D be the intersection of AB and the altitude, as shown in the figure below. If *x* is the length of AD, they should recognize BD = *c* - *x*. Guide students through the following process:



- Use the Pythagorean theorem in ΔBCD: $a^2 = h^2 + (c - x)^2$
- Expand the binomial: $a^2 = h^2 + c^2 - 2cx + x^2$
- Apply the Pythagorean theorem in ΔADC ($x^2 + h^2 = b^2$). What do the expanded binomial yield if $b^2 - x^2$ is substituted for h^2? ($a^2 = b^2 + c^2 - 2cx$).
- What primary trigonometric ratio can be used to determine the altitude? They should recognize that in ΔACD, $\cos A = \frac{x}{b}$, resulting in $x = b\cos A$.
- Substitute $x = b\cos A$ into the equation $a^2 = b^2 + c^2 - 2cx$. What do you notice? ($a^2 = b^2 + c^2 - 2cb\cos A$)

Students can then express the formula in different forms to find the lengths of the other sides of the triangles.

$$b^2 = a^2 + c^2 - 2ac\cos B$$
$$c^2 = a^2 + b^2 - 2ab\cos C$$

The cosine law can be used to determine an unknown side or angle measure in an acute triangle. Continue to encourage students to draw diagrams with both the given and unknown information marked when solving problems. Ask students what information is needed to use the cosine law. They should consider why the cosine law is the only option to find the unknown angle if three sides are known or if two sides and the included angle are known.

General Outcome: Develop spatial sense.

Suggested Assessment Strategies

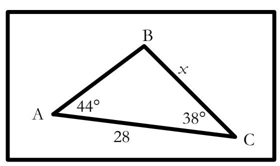
Journal

- Ask students to explain how the Law of Cosines validates the Pythagorean theorem if the included angle is 90°.

(G3.1)

Performance

- In the activity, *Card Sort,* students will sort a set of cards according to a specific strategy labelled sine law, cosine law, Pythagorean theorem, trigonometric ratios. Each card will contain a triangle with information regarding the lengths of the sides and angles. They will have to decide the strategy to solve for the missing angle or side length and place the card in the approriate space on the grid. As students sort the cards, they should discuss their reasons for placing each card into a designated group.

Pythagorean Theorem	Sine Law 
Cosine Law	Primary Trigonometric Ratios

(G3.2, G3.3)

Resources/Notes

Authorized Resource

Principles of Mathematics 11

3.3 Proving and Applying the Cosine Law

SB: pp.144-153
TR: pp.155-162

Web Link

www.k12pl.nl.ca/seniorhigh/introduction/math2201/classroomclips.html

In the **Trigonometry** clip, students participate in the activity *Card Sort.* They discuss the strategy to solve for the missing side lengths and angles of a given triangle.

Geometry

Outcomes

Students will be expected to

G3 Continued...
Achievement Indicators:

G3.2, G3.3 *Continued*

G3.4 *Solve a contextual problem that involves more than one triangle.*

Elaborations—Strategies for Learning and Teaching

When three sides of a triangle are known, students will use the cosine law to find one of the angles. Some students may rearrange the equation to solve for a particular angle. Others may substitute the unknown values into the cosine law and then rearrange the equation to find the angle. It is important for them to recognize they have a choice when trying to find the second angle. They can either use the cosine law or the sine law. Students should notice the third angle can then be determined using the sum of the angles in a triangle.

When solving triangles, encourage students to consider the following questions:

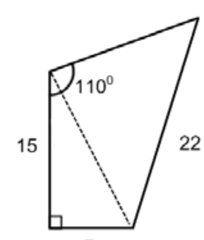
- What is the given information?
- What am I trying to solve for?
- With the given information, should I use the sine law or the cosine law? Is there a choice?
- Which form of the cosine law do I use to solve for an unknown side? Which form do I use to solve for an unknown angle?

If students know two sides and a non-included angle, they can use the cosine law in conjunction with the sine law to find the other side. As an alternative, they could apply the sine law twice. Students have to be exposed to numerous examples to find the method that works best for them.

When working with the cosine law, students sometimes incorrectly apply the order of operations. When asked to simplify $a^2 = 365 - 360\cos 70°$, for example, they often write $a^2 = 5\cos 70°$. To avoid this error, teachers should emphasize that multiplication is to be completed before subtraction.

Students have been exposed to the sine law, the cosine law, the primary trigonometric ratios, the Pythagorean theorem and the sum of the angles in a triangle. They may need to use a strategy or a combination of strategies to solve problems represented by one or more than one triangle. Provide students an example where a playground, in the shape of a quadrilateral, is to be fenced. Ask them to determine the total length of fencing required.

Discussion around various strategies to use as well as the most efficient method, is encouraged. Ask students how changing the right angle in the figure to an acute angle, such as 50°, would affect their strategy.

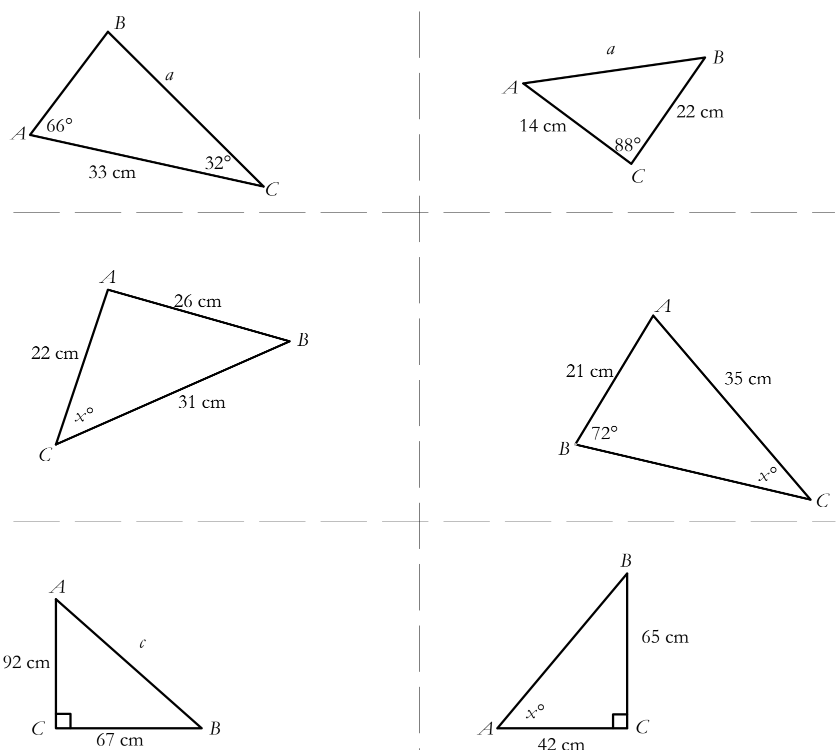


General Outcome: Develop spatial sense.

Suggested Assessment Strategies

Performance

- In the activity, *Four Corners*, students have to think about which method they would use to solve a triangle. Post four signs, one in each corner labelled sine law, cosine law, Pythagorean theorem, trigonometric ratios. Provide each student with one triangle. Instruct the students to make a decision as to which method they would use to find the missing angle or side and to stand in the corner where it is labelled. Once students are all placed, ask them to discuss why their triangle(s) would be best solved using that particular method. Sample triangles are given below:



(G3.2, G3.3)

Resources/Notes

Authorized Resource

Principles of Mathematics 11

3.3 Proving and Applying the Cosine Law

SB: pp.144-153
TR: pp.155-162

Web Link

www.k12pl.nl.ca/seniorhigh/introduction/math2201/classroomclips.html

In the **Trigonometry** clip, students participate in the activity *Four Corners.* They discuss the strategy to solve for the missing side lengths and angles of a given triangle.

3.4 Solving Problems Using Acute Triangles

SB: pp.154-164
TR: pp.163-168

Geometry

Outcomes

Students will be expected to

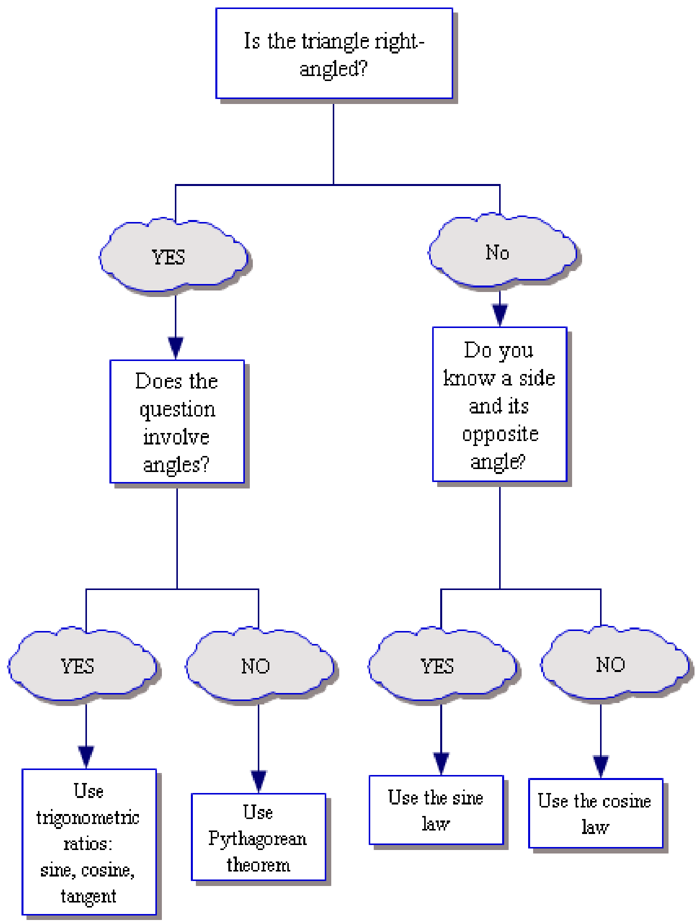
G3 Continued...

Achievement Indicators:

G3.2, G3.3 *Continued*

Elaborations—Strategies for Learning and Teaching

Ask students to create a graphic organizer. When solving triangles, the organizer can guide students as they decide which is the most efficient method to use when solving for an unknown angle and/or side.



Students must recognize that the end result assumes there is enough information remaining in the problem to perform the operation.

General Outcome: Develop spatial sense.

Suggested Assessment Strategies

Presentation

- Ask students to research a real-world problem where the sine law, the cosine law or both is used. They should share the problem with the class and discuss the method for solving. Ask the class to then solve the problem.

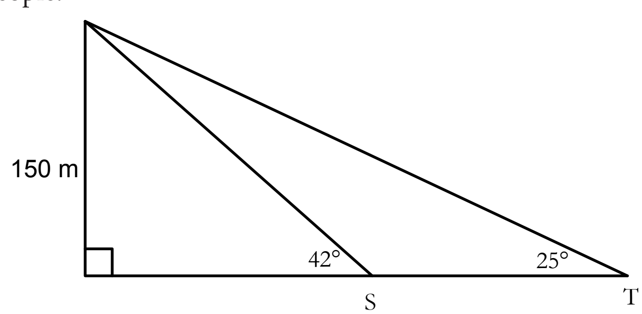
(G3.2, G3.3)

Paper and Pencil

- Ask students what quantities must be known in a triangle before the sine law can be used? the cosine law?

(G3.2, G3.3)

- A smokestack is 150 metres high. Two observers, located at positions S and T, look to the top of the smokestack at angles of elevation of 42° and 25° respectively. How far apart are the two people?



(G3.2, G3.3)

Resources/Notes

Authorized Resource

Principles of Mathematics 11

3.4 Solving Problems Using Acute Triangles

SB: pp.154-164
TR: pp.163-168

Number and Logic

Outcomes

Students will be expected to

NL2 Analyze puzzles and games that involve spatial reasoning, using problem-solving strategies.

[CN, PS, R, V]

Achievement Indicators:

NL2.1 *Determine, explain and verify a strategy to solve a puzzle or to win a game; e.g.,*

- guess and *check*
- *look for a pattern*
- *make a systematic list*
- *draw or model*
- *eliminate possibilities*
- *simplify the original problem*
- *work backward*
- *develop alternative approaches.*

NL2.3 *Identify and correct errors in a solution to a puzzle or in a strategy for winning a game.*

Elaborations—Strategies for Learning and Teaching

Puzzles and games involving spatial reasoning could include the following:

Board Games
(i) Jenga™
(ii) Battleship™
(iii) Checkers™
(iv) Connect Four™
(v) Mancala™

Online Games
(i) Assemble the Square™
(ii) Fifteen Puzzles™
(iii) Tower of Hanoi™
(iv) Rush Hour™
(v) Tetris™
(vi) Nim™
(vii) Mahjong Tiles™
(viii) Tilt™

Paper and Pencil Games
(i) Sprouts™
(ii) Pipelayer™
(iii) Capture™

Gaming Systems
(i) Tanzen™
(ii) Table Tilt - Wii Fit™

When working with puzzles and games, teachers could set up learning stations. Consider the following tips when creating the stations:

- Depending on the nature of the game, some stations may require multiple games, while another station may involve one game that requires more time to play.
- Divide students into groups. After students have had an opportunity to explore the game, ask them to rotate to the next station.

Timing and integration of this outcome should be included in teacher planning throughout the course. Therefore, students can be exposed to three or four games at different times, whether it be at the beginning or end of each unit, or a set "game day". Students could engage in a game when they are finished other work. As students work through the different games and puzzles, they will begin to develop effective strategies for solving the puzzle or game.

General Outcome: Develop number sense and logical reasoning.

Suggested Assessment Strategies

Observation

- Sequential movement puzzles could include:
  - (i) Rush hour
  - (ii) Unblock Me
  - (iii) Lunar Lockout
  - (iv) Flipit
  - (v) Mancala
- Packing puzzles could include:
  - (i) Shape by Shape
  - (ii) Brick by Brick
  - (iii) Tanzen - iPod Touch Application

Students can work through the puzzles on their own either individually or with a partner. They could record their progress on a sheet such as the one shown below.

Puzzle	Solved?	Comments/Hints
1		
2		
3		
4		
5		

(NL2.1, NL2.3)

Resources/Notes

Authorized Resource

Principles of Mathematics 11

3.4: Solving Problems Using Acute Triangles

SB: pp.165
TR: pp.168-169

Number and Logic

Outcomes

Students will be expected to

NL2 Continued...

Achievement Indicators:

NL2.1, NL2.3 *Continued*

Elaborations—Strategies for Learning and Teaching

There may be situations where students are able to play the game and solve problems but are unable to determine a winning strategy. Teachers could participate with the group and think through the strategies out loud so the group can hear the reasoning for selected moves. Ask the groups' opinions about moves in the game and facilitate discussions around each of the other players' moves and strategies.

As students work through the games and puzzles, teachers should keep a checklist of the games and puzzles each student is working on. This would be a good opportunity for students to work in groups where each has been exposed to a different game or puzzle. Ask the group of students to participate in the following activity:

- Explain the rules of the game in your own words to other students.
- Give a brief demonstration.
- Explain the strategy you tried in solving the puzzle or playing the game.
- What general advice would you give to other students trying to solve the puzzle or play the game?
- What did you do when you got stuck?

General Outcome: Develop number sense and logical reasoning.

Suggested Assessment Strategies

Journal

- Ask students to write about their thought processes while solving the puzzle. Which puzzles did they find interesting and why?

  Assist students using prompts like the following;

  (i) What did you do first?

  (ii) How did you reach that answer?

  (iii) Did you notice any patterns?

  (iv) What were some challenges when solving the puzzle?

  (v) What strategies did you use to solve the puzzle?

(NL2.1,NL2.3)

Resources/Notes

Authorized Resource

Principles of Mathematics 11

3.4: Solving Problems Using Acute Triangles

SB: pp.165
TR: pp.168-169

Radicals

Suggested Time: 18 Hours

Unit Overview

Focus and Context

In this unit, students will simplify radicals with a numerical radicand. They will compare and order radical expressions and express the radicals as either mixed or entire. Students will then work with radicals with variable radicands and identify restrictions on the values for the variable in the radical expression.

Students will solve problems that involve operations on radicals and radical expressions with numerical and variable radicands (limited to square roots). When working with division, they will only be responsible for a monomial denominator.

Students will then solve problems that involve radical equations, limited to square roots and cube roots. They will either square or cube both sides of the equation and solve the resulting linear equation. Students will then verify that the values determined in solving a radical equation are roots of the equation.

Outcomes Framework

GCO

Develop algebraic reasoning and number sense.

SCO NL3

Solve problems that involve operations on radicals and radical expressions with numerical and variable radicands (limited to square roots).

SCO NL4

Solve problems that involve radical equations (limited to square roots or cube roots).

GCO

Develop number sense and logical reasoning.

SCO NL2

Analyze puzzles and games that involve spatial reasoning, using problem-solving strategies.

Process Standards Key

[C]	Communication	[PS]	Problem Solving
[CN]	Connections	[R]	Reasoning
[ME]	Mental Mathematics and Estimation	[T]	Technology
		[V]	Visualization

SCO Continuum

Mathematics 1201	Mathematics 2201	Mathematics 3201
Algebra and Number	Number and Logic	
AN1 Demonstrate an understanding of factors of whole numbers by determining the: • prime factors • greatest common factor • least common multiple • square root • cube root. [CN, ME, R]	NL3 Solve problems that involve operations on radicals and radical expressions with numerical and variable radicands (limited to square roots). [CN, ME, PS, R]	
	NL4 Solve problems that involve radical equations (limited to square roots or cube roots). [C, PS, R]	
		Logical Reasoning
	NL2 Analyze puzzles and games that involve spatial reasoning, using problem-solving strategies. [CN, PS, R, V]	LR1 Analyze puzzles and games that involve numerical and logical reasoning, using problem-solving strategies. [CN, ME, PS, R]

Number and Logic

Outcomes

Students will be expected to

NL3 Solve problems that involve operations on radicals and radical expressions with numerical and variable radicands (limited to square roots).

[CN, ME, PS, R]

Achievement Indicators:

NL3.1 *Express a mixed radical with a numerical radicand as an entire radical.*

NL3.2 *Compare and order radical expressions with numerical radicands.*

NL3.3 *Express an entire radical with a numerical radicand as a mixed radical.*

Elaborations—Strategies for Learning and Teaching

In Grade 9, students determined the square root of a perfect square and worked with benchmarks to approximate the square roots of non-perfect square rational numbers (9N5, 9N6). In Mathematics 1201, students worked with mixed and entire radicals, limited to numerical radicands (AN2). The notation $\sqrt[n]{x}$ was introduced, where the index of the radical was a maximum of 5.

In this unit, students will simplify radical expressions with numerical and variable radicands and will add, subtract, multiply and divide these expressions. When working with division, students will only be responsible for a monomial denominator. They will also identify values of the variable for which the radical expression is defined.

Students will simplify radicals with numerical and variable radicands and then perform operations with numerical and variable radicands. Teachers may wish to simplify and perform the four operations with numerical radicands before moving on to variable radicands.

In Mathematics 1201, students expressed a radical as either a mixed or entire radical with numerical radicands. Review this concept with students and reinforce that if radicals have the same index, the radicands can be compared. It is helpful to rearrange the mixed radical as an entire radical for the purpose of ordering and estimation without the use of technology. When given two right triangles, ask students to determine which hypotenuse length is greater, $3\sqrt{5}$ or $4\sqrt{3}$. Although students could use a calculator to approximate the length, the focus here is to rewrite the numerical radicals in equivalent forms and make a comparison.

Part of this unit is a review of topics in Mathematics 1201, with the exception of the introduction to secondary roots. Refer to the Roots and Powers unit in the Mathematics 1201 Curriculum Guide for further information regarding teaching strategies and common student errors (AN2.6, AN2.7, AN2.8).

Students should have some exposure to principal and secondary square roots. Ask students to respond to the following:

- Choose any two opposite numbers.
- Square the numbers. What do you notice?
- What does this imply about the square root of a positive number?

General Outcome: Develop number sense and logical reasoning.

Suggested Assessment Strategies

Paper and Pencil

- Ask students to write the following as a mixed or entire radical:
  (i) $\sqrt[3]{54}$
  (ii) $4\sqrt[3]{2}$
  (iii) $3\sqrt[5]{2}$

(NL3.1, NL3.3)

- Ask students to arrange the following from least to greatest:

$$2\sqrt{5}, \sqrt{59}, 4\sqrt[3]{2}, \sqrt[4]{48}$$

(NL3.2)

Performance

- Prepare a set of cards with a variety of radicals with numerical radicands. Give each student one card. Place them in teams. Students must compare their cards and arrange themselves so that they are in order (ascending or descending). The team who correctly orders the cards first is the winner.

(NL3.2)

- Students can play the *Radical Matching Game* in groups of two. Give students a deck of cards containing pairs that display equivalent mixed radicals and entire radicals. All cards should be placed face down on the table. The first student turns over 2 of the cards, looking for a pair. If they get a pair, they remove the cards and go again. If the overturned cards do not form a pair, it is the other player's turn. The player with the most matches at the end of the game wins.

(NL3.1, NL3.3)

Resources/Notes

Authorized Resource

Principles of Mathematics 11

Note: The resource first simplifies numerical radicands and solves problems that involve operations on these expressions before moving on to radicals with variable radicands.

4.1: Mixed and Entire Radicals
Student Book (SB): pp.176-183,
Teacher Resource (TR): pp.195-199

4.4: Simplifying Algebraic Expressions Involving Radicals
Student Book(SB): pp.204-213
Teacher Resource(TR): pp.211-215

Web Link

The information regarding radicals can be found in the Mathematics 1201 Curriculum Guide.

www.ed.gov.nl.ca/edu/k12/curriculum/guides/index.html.

Number and Logic

Outcomes

Students will be expected to

NL3 Continued...

Achievement Indicators:

NL3.1, NL3.2

NL3.3 *Continued*

NL3.4 *Identify values of the variable for which the radical expression is defined.*

Elaborations—Strategies for Learning and Teaching

Students should recognize that every positive number has two roots. For example, the square root of 49 is 7 since $7^2 = 49$. Likewise $(-7)^2 = 49$ so -7 is also a square root of 49. Students will be exposed to $\sqrt{49} = 7$ as the principal square root and $-\sqrt{49} = -7$ as the secondary square root. Although students will compare principal and secondary square roots, it is equally important for them to understand why it makes sense to only use the principal square root in certain situations. For example, if students are using the Pythagorean theorem to calculate the length of a leg of a right triangle, length is a positive number, so the square root must also be positive. This concept will be explored further when students start work with quadratics later in this course.

In Mathematics 1201, students simplified radicals with numerical radicands. They also determined the domain of a variety of functions (RF1). This is their first exposure to simplifying expressions with variable radicands, limited to square roots. They will write the restrictions on the variable and then write the expression in its simplest form.

The domain of a square root function is limited to values for which the function has meaning. Students will state restrictions on the variable since the square root of a negative number does not belong to the real number system. Ask them to complete the following chart and make predictions on the restrictions of each variable. Use leading questions such as the following to promote the discussion:

- What values of *x* were undefined? What values of *x* were defined?
- Is the restriction different if the radical expression is in the denominator?
- How could solving inequalities help when determining the restriction?

x	$\sqrt{x}$	$\frac{1}{x}$	$\frac{1}{\sqrt{x}}$		$\sqrt{x-1}$	$\sqrt{x+1}$	$\frac{1}{\sqrt{x-1}}$	$\frac{1}{\sqrt{x+1}}$
-3								
-2								
-1								
0								
1								
2								
3								

General Outcome: Develop number sense and logical reasoning.

Suggested Assessment Strategies

Performance

- In groups of two, ask students to play the game *Restrict It*. Cards will be created displaying radical expressions and their restrictions. The cards with radical expressions will be placed on one wall of the classroom and their associated restriction can be placed on the opposite or adjacent wall. Members of the group will take a card with a restriction and place it with its correct expression. There could be more radical expressions than restrictions to make this game more challenging.

(NL3.4)

- Ask half the class to create radical expressions on cards. The other half of the class will create restrictions of their choice. When the students trade their cards with another classmate, students who receive a restriction will write a possible radical expression, while students who receive a radical expression will write the restriction.

(NL3.4)

Interview

- Ask students to show and explain why the domain of $\sqrt{2x-5}$ is $x \geq \frac{5}{2}$, while the domain of $\frac{1}{\sqrt{2x-5}}$ is $x > \frac{5}{2}$.

(NL3.4)

Resources/Notes

Authorized Resource

Principles of Mathematics 11

4.1: Mixed and Entire Radicals
SB: pp.176-183,
TR: pp.195-199

4.4: Simplifying Algebraic Expressions Involving Radicals
SB: pp.204-213
TR: pp.211-215

Number and Logic

Outcomes

Students will be expected to

NL3 Continued...

Achievement Indicators:

NL3.4 *Continued*

NL3.5 *Express an entire radical with a variable radicand as a mixed radical.*

Elaborations—Strategies for Learning and Teaching

The goal is for students to recognize that the expression under the radical must be greater than or equal to zero. That is, they must identify the values of *x* for which the radical is a real number. When *x* is in the denominator of a rational expression, there is an additional constraint. In this case, students must also identify values of *x* where the expression is not equal to zero.

In Mathematics 1201, students expressed a radical as a power with rational exponents (AN3). The focus here will be exclusive to the square root of a radical with variable radicands (i.e., $\sqrt{x^n} = x^{\frac{n}{2}}$).

When asked to simplify $\sqrt{x^2}$, students may initially conclude that $\sqrt{x^2} = x$. Prompt discussion using the following questions:

- Does this happen with all positive values of *x*?
- Does this happen with all negative values of *x*?

Ask students to simplify $\sqrt{(-5)^2}$. The following is a sample of possible student answers:

$\sqrt{(-5)^2} = (-5)^{\frac{2}{2}} = (-5)^1 = -5$	$\sqrt{(-5)^2} = \sqrt{25} = 5$

This is a great opportunity for discussion around the correct answer when the value of *x* is negative. Some students may challenge the incorrect solution provided in column 1 above. It is important for students to recognize that when *a* is a negative number, $a^{\frac{m}{n}}$ is not defined because it is not possible to define such expressions consistently. Students can compare this value when *n* is even to when it is odd. (i.e., $(-4)^{\frac{1}{2}} = \sqrt{-4}$ is undefined under the set of real numbers but $(-8)^{\frac{1}{3}} = \sqrt[3]{-8}$ is defined).

Although it is common for students to replace $\sqrt{x^2}$ with *x*, students must recognize that this is correct when $x \geq 0$. They should also realize that $\sqrt{x^2}$ is equivalent to $-x$ when $x \leq 0$. Consider the following:

When $x = 4$:

$$\sqrt{x^2} = \sqrt{(4)^2}$$
$$\sqrt{x^2} = \sqrt{16}$$
$$\sqrt{x^2} = 4$$
$$\therefore \sqrt{x^2} = x$$

When $x = -4$:

$$\sqrt{x^2} = \sqrt{(-4)^2}$$
$$\sqrt{x^2} = \sqrt{16}$$
$$\sqrt{x^2} = 4$$
$$\sqrt{x^2} = -x$$
$$\therefore \sqrt{x^2} \neq x$$

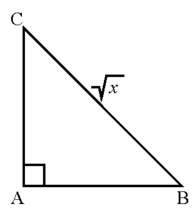
General Outcome: Develop number sense and logical reasoning.

Suggested Assessment Strategies

Journal

- Ask students to respond to the following:
  (i) Your friend stated that since $3^2 = 9$ and $(-3)^2 = 9$, the $\sqrt{x^2}$ is always $\pm x$. Ask students to use specific examples to show whether they agree or disagree with this statement.
  (NL3.4, NL3.5)

  (ii) The length of $\overline{BC} = \sqrt{x}$. Ask students why $x \geq 0$ is not applicable in this situation.



(NL3.4)

Resources/Notes

Authorized Resource

Principles of Mathematics 11

4.1: Mixed and Entire Radicals
SB: pp.176-183
TR: pp.195-199

4.4: Simplifying Algebraic Expressions Involving Radicals
SB: pp.204-213
TR: pp.211-215

Number and Logic

Outcomes

Students will be expected to

NL3 Continued...

Achievement Indicators:

NL3.5 *Continued*

Elaborations—Strategies for Learning and Teaching

Introduce students to the absolute value notation and what it represents. The use of the absolute value ensures that the principal square root is always represented. Therefore, $\sqrt{x^2} = |x|$. Once this idea has been established, students can assume, for the sake of simplicity, all variables are positive. Therefore, $\sqrt{x^2} = x$.

Examples such as the following will help students to recognize the pattern when simplifying radicals with variable radicands.

	Greatest Perfect Square	Prime Factorization
$\sqrt{125}$	$\sqrt{25 \times 5} = 5\sqrt{5}$	$\sqrt{5 \times 5 \times 5} = 5\sqrt{5}$
$\sqrt{x^3}$	$\sqrt{x^2 \times x} = x\sqrt{x}$	$\sqrt{x \times x \times x} = x\sqrt{x}$
$\sqrt{x^4}$	$\sqrt{x^2 \times x^2} = x \times x = x^2$	$\sqrt{x \times x \times x \times x} = x \times x = x^2$
$\sqrt{x^5}$		
$\sqrt{x^6}$		

Students should recognize that when the exponent is even, the resulting expression is a polynomial of the form x^n. When the exponent is odd however, the resulting expression is a mixed radical of the form $x^n\sqrt{x}$. As students revisit the restrictions on the domain, encourage them to discuss what restrictions exist when there is an even exponent versus an odd exponent.

This is an important foundation for students as it leads into the introduction of monomial expressions under the root sign and the simplification of mixed radicals. Some examples include: $\sqrt{4x^2}$, $\sqrt{54x^5}$, $4\sqrt{8x^3y^4}$.

NL3.6 *Perform one or more operations to simplify radical expressions with numerical or variable radicands.*

Students should work with radicals that contain numerical radicands and coefficients before moving on to radicals that contain variables. They will add and subtract radicals, limited to square roots, before they move into multiplication and division by a monomial.

This could be introduced by asking students to add $2x + 3x$. Use leading questions such as the following:

- What process is involved in the addition of monomials?
- How is this expression similar to $2\sqrt{7} + 3\sqrt{7}$?
- What are like radicals?
- How can this process be applied to radicals?

The goal is for students to realize that adding and subtracting radical expressions is comparable to combining variable expressions with like terms. It is necessary for the radical to have the same index and the same radicand.

General Outcome: Develop number sense and logical reasoning.

Suggested Assessment Strategies

Paper and Pencil

- Ask students to create an expression with at least four different radicals that simplify to $5\sqrt{2}-8\sqrt{3}$. Can they create more than one expression? Students should share their results with the rest of the class.

(NL3.6)

Performance

- Ask students to participate in the activity *Sticky Bars*. Present students with a selected response question where they could be expected to convert an entire radical with a variable radicand to a mixed radical. The answer is anonymously recorded on a post it note and submitted to the teacher. Ask a student volunteer to arrange the sticky notes on the wall or whiteboard as a bar graph representing the different student responses. They should explain why some student repsonses may have led to certain answers.

(NL3.5)

Resources/Notes

Authorized Resource

Principles of Mathematics 11

4.1: Mixed and Entire Radicals
SB: pp.176-183,
TR: pp.195-199

4.4: Simplifying Algebraic Expressions Involving Radicals
SB: pp.204-213
TR: pp.211-215

Web Link

This program allows students to match the entire radical with the mixed radical involving numerical and variable radicands.

www.quia.com/mc/678599.html.

4.2: Adding and Subtracting Radicals
SB: pp.184-190
TR: pp.200-202

4.4: Simplifying Algebraic Expressions Involving Radicals
SB: pp.204-213
TR: pp.211-215

Number and Logic

Outcomes

Students will be expected to

NL3 Continued...
Achievement Indicator:

NL3.6 *Continued*

Elaborations—Strategies for Learning and Teaching

The distributive property can also be applied when simplifying sums and differences of radical expressions. Ask students how to rewrite $2x + 3x$ in another form, namely $(2+3)x$. Similarly, $2\sqrt{7}+3\sqrt{7}$ can be expressed as $(2+3)\sqrt{7} = 5\sqrt{7}$. This would be a good opportunity to check students' understanding by asking them to express $-4\sqrt{3}$ as the sum of two like radicals.

It is important for students to recognize that, even when adding or subtracting, the solution can require further simplifying. When subtracting the expression $3\sqrt{8}-7\sqrt{8}$, for example, the solution $-4\sqrt{8}$ can be simplified to $-8\sqrt{2}$. It would be good practice to simplify the radical before like terms are combined. This is especially beneficial when working with large numerical radicands. Students should also be exposed to examples such as $2\sqrt{18}+3\sqrt{50}-5\sqrt{2}$ where it is necessary to simplify one or more radicals in order to complete the addition and/or subtraction operations.

Common errors occur when adding or subtracting radicals. For example, when asked to add $4+2\sqrt{3}$ students may write $6\sqrt{3}$.
Students should be encouraged to check their answers by expressing the sum/difference using the distributive property. Another common error occurs when they incorrectly apply the operations of addition and subtraction to the radicands. The value of $31\sqrt{3}+17\sqrt{3}$, for example, does not equal $48\sqrt{6}$. This error may occur more often once students have been introduced to the multiplication of radicals. It may benefit some students to interpret each radical as an example of length. Using a calculator or a number line, students can explore the length of $\sqrt{3}+\sqrt{3}$ and see that it is not the same length as $2\sqrt{6}$.

Student learning will continue with addition and subtraction involving variable radicands. This would be an opportunity to ask students to discuss questions such as the following:

- How is adding and subtracting algebraic expressions with radicals similar to adding and subtracting numerical expressions with radical values? Use $\sqrt{8x^3}-4\sqrt{2x}$ and $2\sqrt{45}-6\sqrt{20}$ to explain your reasoning.
- What strategy did you use?

General Outcome: Develop number sense and logical reasoning.

Suggested Assessment Strategies

Performance

- In groups of two or three, ask students to play the *Concentration Game*. Give students various addition and/or subtraction expressions with radicals and their simplified solutions on pairs of cards. The cards are placed face down. A student flips two cards to determine if they are equivalent. They should be able to explain why they are or are not equivalent. If a student gets a match, they keep playing. If not, the next person plays until all pairs are compiled.

(NL3.6)

- Create centres in the classroom containing worked solutions of simplifying radical expressions with numerical or variable radicands. Students will participate in a carousel activity where they are asked to move throughout the centres to identify and correct errors. Samples are shown below:

(i) $25\sqrt{5} + 13\sqrt{5}$

$= 38\sqrt{10}$

(ii) $\sqrt{18x^3} + 2\sqrt{8x^3}$

$= 3\sqrt{2x^3} + 4\sqrt{2x^3}$

$= 7\sqrt{4x^6}$

$= 14x^3$

(NL3.6)

Paper and Pencil

- Ask students to simplify each of the following:

(i) $2\sqrt{18} + 9\sqrt{7} - \sqrt{63}$

(ii) $4\sqrt{x} + 2\sqrt{x} - 7\sqrt{x}$

(iii) $6\sqrt{32x^3} - 5\sqrt{8x^3} + 3\sqrt{2x^3}$ (NL3.6)

Resources/Notes

Authorized Resource

Principles of Mathematics 11

4.2: Adding and Subtracting Radicals
SB: pp.184-190
TR: pp.200-202

4.4: Simplifying Algebraic Expressions Involving Radicals
SB: pp.204-213
TR: pp.211-215

Number and Logic

Outcomes

Students will be expected to

NL3 Continued...
Achievement Indicator:

NL3.6 *Continued*

Elaborations—Strategies for Learning and Teaching

Real-world examples, such as the following, provide a good opportunity to discuss the use of exact and approximate answers.

- The voltage V required for a circuit is given by $V=\sqrt{PR}$ where P is the power in watts and R is the resistance in ohms. How many more volts are needed to light a 100-W bulb than a 75-W bulb if the resistance for both is 100 ohms?

Similar to adding and subtracting radicals, students will multiply and divide radicals beginning with numerical radicands. To demonstrate the multiplication property of radicals, reiterate the relationship between a radical and a power with rational exponents.

$$3^{\frac{1}{2}}\times 5^{\frac{1}{2}}=(3\times 5)^{\frac{1}{2}}=15^{\frac{1}{2}}\Leftrightarrow \sqrt{3}\times\sqrt{5}=\sqrt{3\times 5}=\sqrt{15}$$

In Mathematics 1201, students applied the laws of exponents to rational exponents (AN3).

Encourage students to look for a pattern through the use of several examples. The multiplicative property of radicals can then be introduced to students, $\sqrt{a}\times\sqrt{b}=\sqrt{ab}$ where $a\geq 0$ and $b\geq 0$. This property can also be used to discuss why two radicals with the same index can be multiplied. Although students were exposed to this property in Mathematics 1201, it was used exclusively for the purpose of expressing a radical as a mixed or entire radical (AN2).

Students should be given an opportunity to further explore the product rule for radicals (i.e., $c\sqrt{a}\times d\sqrt{b}=cd\sqrt{ab}$), and the commutative property. Ask them to rewrite the expression $2(3^{\frac{1}{2}})\times 5(6^{\frac{1}{2}})$ using radicals to generalize a pattern. Students' workings may vary but the result should be the same. Consider the following sample:

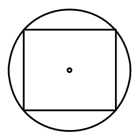
$2(3^{\frac{1}{2}})\times 5(6^{\frac{1}{2}})$	$2(\sqrt{3})\times 5(\sqrt{6})$
$(2\times 5)(3^{\frac{1}{2}}\times 6^{\frac{1}{2}})$	$(2\times 5)\left(\sqrt{3}\times\sqrt{6}\right)$
$(10)(3\times 6)^{\frac{1}{2}}$	$(10)\left(\sqrt{3\times 6}\right)$
$10(18)^{\frac{1}{2}}$	$10\sqrt{18}$

General Outcome: Develop number sense and logical reasoning.

Suggested Assessment Strategies

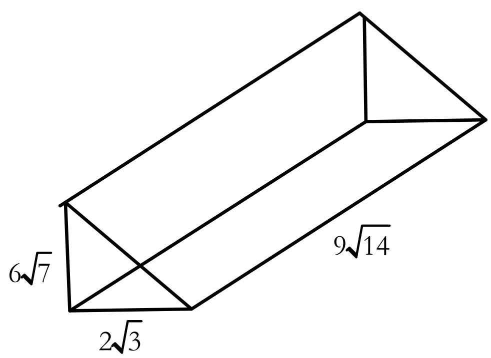
Paper and Pencil

- The radius of a circle is $3\sqrt{5}$ m. If a square is inscribed in a circle, ask students to:
  - (i) Determine the exact length of the diagonal of the square.
  - (ii) Determine the exact perimeter of the square.



(NL3.6)

- Given the right triangular prism, ask students to:
  - (i) Determine the volume.
  - (ii) Determine the surface area.



(NL3.6)

Resources/Notes

Authorized Resource

Principles of Mathematics 11

4.3: Multiplying and Dividing Radicals
SB: pp.191-201
TR: pp.203-209

4.4: Simplifying Algebraic Expressions Involving Radicals
SB: pp.204-213
TR: pp.211-215

Number and Logic

Outcomes

Students will be expected to

NL3 Continued...

Achievement Indicator:

NL3.6 *Continued*

Elaborations—Strategies for Learning and Teaching

Advise students that they are less likely to make simplification errors if they simplify radicals before multiplying. An example such as the following could be used to illustrate the two methods:

Multiply First: $\sqrt{80}\times\sqrt{12}=\sqrt{960}=8\sqrt{15}$

Simplify first: $\sqrt{80}\times\sqrt{12}=4\sqrt{5}\times2\sqrt{3}=8\sqrt{15}$

Ask students which method they prefer and why.

Explore with students what happens when the radicand is the same in each expression. Consider the example $7\sqrt{2}\times3\sqrt{2}$. Allow students time to try this example and ask them the following questions:

- Were you surprised that the answer was 42?
- Why is the product of two identical radical expressions an integer even though each factor is an irrational number?

Students will also multiply radicals using the distributive property. It may be helpful to walk them through the similarities between multiplying radical expressions and multiplying polynomials. The steps involved in multiplying $4x(x+1)$ or $(2x-1)(3x+5)$, for example, include applying the distributive property and combining like terms. A similar process is used to multiply radical expressions such as $(8\sqrt{2}-5)(9\sqrt{5}+6\sqrt{10})$.

In order to avoid common errors, this would be a good opportunity to reinforce the commutative and associative property of multiplication. When multiplying $\sqrt{5}\times3$, for example, students may write $\sqrt{15}$. They may understand their error if they apply the commutative property to rewrite the expression as $3\sqrt{5}$. Another error occurs when students are asked to multiply an expression such as $3\sqrt{5}\times\sqrt{6}$ and their result is $\sqrt{90}$. This can be avoided if the associative property is used to re-order the expression as $3\left(\sqrt{5}\times\sqrt{6}\right)=3\sqrt{30}$.

General Outcome: Develop number sense and logical reasoning.

Suggested Assessment Strategies

Paper and Pencil

- Ask students to simplify each of the following:

  (i) $(\sqrt{2}+\sqrt{6})^2$

  (ii) $(3\sqrt{8}-4)(2+7\sqrt{3})$

  (iii) $(\sqrt{20}+\sqrt{24})(3\sqrt{12}-5\sqrt{32})$

(NL3.6)

Performance

- Ask students to participate in *Commit and Toss.* Provide students with a selected response problem as shown below. They anonymously commit to an answer and provide a justification for the answer they selected. Students crumble their solutions into a ball and toss the papers into a basket. Once all papers are in the basket, ask students to reach in and take one out. Ask students to then move to the corner of the room designated to match their selected response. In their respective corners, they should discuss the similarities or differences in the explanations provided and report back to the class.

Express $(\sqrt{3}-\sqrt{2})^2$ in simplest form. (A) 1 (B) $5-2\sqrt{6}$ (C) $1-2\sqrt{3}$ (D) 5 Explain your reasoning:

(NL3.6)

Resources/Notes

Authorized Resource

Principles of Mathematics 11

4.3: Multiplying and Dividing Radicals
SB: pp.191-201
TR: pp.203-209

4.4: Simplifying Algebraic Expressions Involving Radicals
SB: pp.204-213
TR: pp.211-215

Web Link

www.k12pl.nl.ca/seniorhigh/introduction/math2201/classroomclips.html

The **Radical Expressions** clip demonstrates students performing operations to simplify a radical expression using the activity *Commit and Toss*.

Number and Logic

Outcomes

Students will be expected to

NL3 Continued...

Achievement Indicators:

NL3.6 *Continued*

Elaborations—Strategies for Learning and Teaching

It is beneficial to have students analyze solutions that contain errors. This reinforces the importance of recording solution steps rather than only giving a final answer. When multiplying radical expressions, students sometimes incorrectly apply the distributive property and/or the order of operations. Teachers could arrange centres in the classroom where groups of students visit each centre to identify incorrect solutions, why the error might have occurred and how they can be corrected. Some examples may include the following:

(i) $(2\sqrt{3})(4\sqrt{5}) = 8 + 2\sqrt{5} + 4\sqrt{3} + \sqrt{15}$

(ii) $(2\sqrt{3} + 5\sqrt{2})^2 = 4\sqrt{9} + 25\sqrt{4}$

(iii) $2 - 5\sqrt{7}(3 + 4\sqrt{7}) = (2 - 5\sqrt{7})(3 + 4\sqrt{7})$

Students will explore how multiplying algebraic expressions with radicals is similar to multiplying numerical expressions with radical values. Ask them to consider the examples $-4\sqrt{12} \times -2\sqrt{18}$ and $(-4\sqrt{x}) \times (-2\sqrt{x^2})$, where $x \geq 0$, to explain their reasoning.

- What strategy did you use?
- When is it necessary to use the distributive property to multiply expressions that contain radicals? Create an example and show the solution.

NL3.7 *Rationalize the monomial denominator of a radical expression.*

Division of radicals and the rationalization of the denominator are new concepts for students. The denominator will be restricted to monomials.

The rules of exponents should be integrated when introducing the division of radicals. For example:

$$\left(\frac{4}{9}\right)^{\frac{1}{2}} = \frac{4^{\frac{1}{2}}}{9^{\frac{1}{2}}} \Leftrightarrow \sqrt{\frac{4}{9}} = \frac{\sqrt{4}}{\sqrt{9}}$$

In other words, the quotient rule of radicals states that the square root of a quotient is the quotient of the square roots.

$$\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}} \text{ where } a \geq 0, b > 0$$

General Outcome: Develop number sense and logical reasoning.

Suggested Assessment Strategies

Paper and Pencil

- Ask students to simplify each of the following
  (i) $\left(3\sqrt{x}+2\right)\left(3-5\sqrt{x}\right)$

  (ii) $\left(-3\sqrt{x}\right)\left(6\sqrt{x^3}\right)$

(NL3.6, NL3.7)

Performance

- Ask students to select one of the operations involving radicals studied in this unit and have them create an educational brochure/poster on the chosen operation. Their brochure/poster should include the following features:
  (i) An explanation of the steps involved in performing the operation.
  (ii) Examples which illustrate their understanding of the operation they have chosen.
  (iii) Common errors for that particular operation.

(NL3.6, NL3.7)

Resources/Notes

Authorized Resource

Principles of Mathematics 11

4.3: Multiplying and Dividing Radicals
SB: pp.191-201
TR: pp.203-209

4.4: Simplifying Algebraic Expressions Involving Radicals
SB: pp.204-213
TR: pp.211-215

Number and Logic

Outcomes

Students will be expected to

NL3 Continued...
Achievement Indicator:

NL3.7 *Continued*

Elaborations—Strategies for Learning and Teaching

Students should be exposed to a variety of cases when simplifying radicals with fractions. Include examples such as the following:

$$\sqrt{\frac{25}{4}} = \frac{\sqrt{25}}{\sqrt{4}} = \frac{5}{2} \qquad \frac{\sqrt{12}}{\sqrt{6}} = \sqrt{\frac{12}{6}} = \sqrt{2} \qquad \sqrt{\frac{12}{5}} = \frac{\sqrt{12}}{\sqrt{5}} = \frac{2\sqrt{3}}{\sqrt{5}}$$

As students simplify radicals such as these, they should ask themslves the following questions:

- Is the denominator a perfect root?
- Can the numerator and denominator divide into a rational number?
- Will the denominator have a radical when simplified?

Students will develop a strategy for converting a fraction that has radicals in its denominator into an equivalent fraction with no radicals in the denominator. Rationalizing the denominator provides a standard notation for expressing results. Using an example such as $\frac{2\sqrt{3}}{\sqrt{5}}$, ask them what expression would eliminate the radical when multiplied onto the denominator.

Examples such as $\frac{2\sqrt{3}}{7\sqrt{5}}$, where the denominator is a mixed radical should also be included. The initial tendency may be to multiply the numerator and denominator by $7\sqrt{5}$. Encourage students to think about what they would have to multiply by to rationalize the denominator. Allow students time to experiment with this. Although it is not the most efficient strategy, it is correct. However, the resulting expression will have to be simplified. They should realize multiplying by $\frac{\sqrt{5}}{\sqrt{5}}$ will rationalize the denominator.

Students should first be exposed to expressions with entire radicals in both the numerator and denominator. This should then be extended to include mixed radicals and examples where there is more than one term in the numerator. It is the students' choice whether they simplify before or after they rationalize the denominator. When working with larger numbers, however, simplifying first would allow them to work with smaller numbers.

General Outcome: Develop number sense and logical reasoning.

Suggested Assessment Strategies

Paper and Pencil

- Ask students to play the following puzzle game involving multiplying and dividing radicals. They will match each side with the corresponding answer on another puzzle piece.

Top: $12\sqrt{2}$ Left: $\frac{20+2\sqrt{15}}{15}$ Right: $\sqrt{\frac{100}{2}}$ Bottom: $(5\sqrt{2}-3\sqrt{12})(2\sqrt{2}+4\sqrt{3})$	Top: $\frac{\sqrt{24}}{\sqrt{6}}$ Left: $5\sqrt{2}$ Right: $(5\sqrt{5})(3\sqrt{5})$ Bottom: 108	Top: $\sqrt{9}+\sqrt{15}$ Left: 75 Right: $\frac{\sqrt{80}+2\sqrt{3}}{3\sqrt{5}}$ Bottom: $\sqrt{63}\div\sqrt{7}$
Top: $8\sqrt{6}-52$ Left: $45+3\sqrt{3}$ Right: $(2\sqrt{3})(3\sqrt{5})$ Bottom: $\frac{4\sqrt{3}}{\sqrt{2}}$	Top: $(12\sqrt{3})(3\sqrt{3})$ Left: $6\sqrt{15}$ Bottom: $(\sqrt{3}+\sqrt{2})^2$	Top: 3 Left: $3\sqrt{8}(2\sqrt{5}-4\sqrt{2})$ Right: $3(15+\sqrt{3})$ Bottom: $\sqrt{6}\times 4$
Top: $2\sqrt{6}$ Left: 5 Right: $\frac{6\sqrt{24}}{2\sqrt{3}}$ Bottom: $\frac{\sqrt{48}}{\sqrt{2}}$	Top: $5+2\sqrt{6}$ Left: $6\sqrt{2}$ Right: $3\sqrt{3}+15$ Bottom: $15\sqrt{3}$	Top: $4\sqrt{6}$ Left: $\sqrt{3}(3+\sqrt{5})$ Right: -52 Bottom: $\frac{4\sqrt{2}+\sqrt{6}}{2}$

(NL3.6,NL3.7)

Resources/Notes

Authorized Resource

Principles of Mathematics 11

4.3: Multiplying and Dividing Radicals
SB: pp.191-201
TR: pp.203-209

4.4: Simplifying Algebraic Expressions Involving Radicals
SB: pp.204-213
TR: pp.211-215

Number and Logic

Outcomes

Students will be expected to

NL3 Continued...

Achievement Indicator:

NL3.7 *Continued*

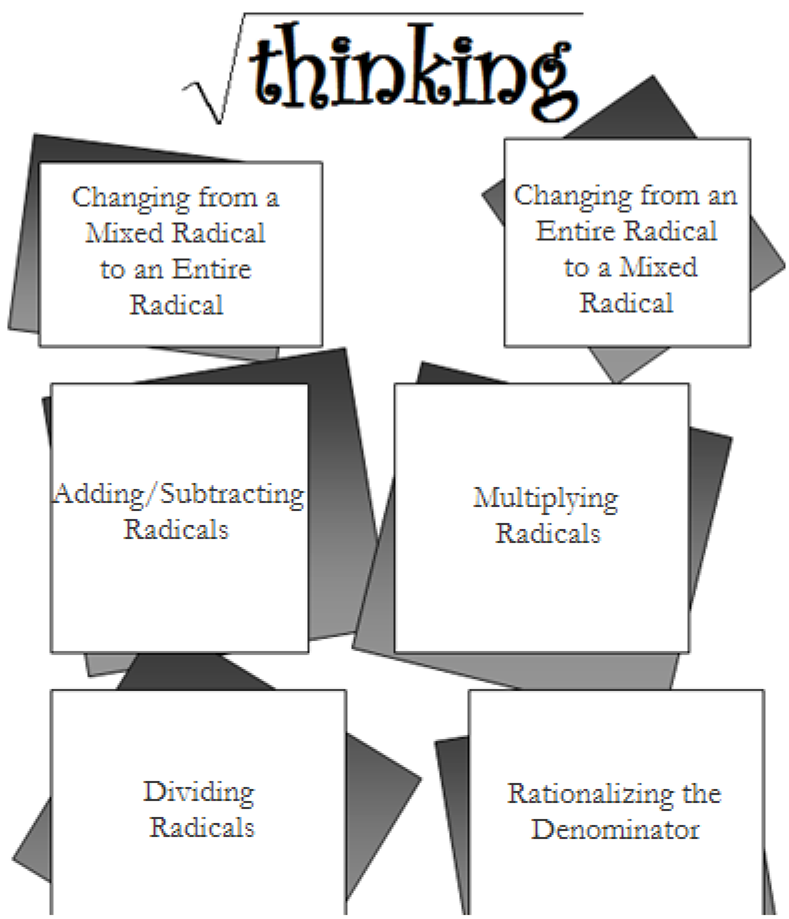
Elaborations—Strategies for Learning and Teaching

Division involving variable monomial denominators could be approached in a similar fashion using examples that parallel numerical ones as illustrated below:

$$\sqrt{\frac{1}{8}} = \frac{1}{2\sqrt{2}} = \frac{1}{2\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{2\times 2} = \frac{\sqrt{2}}{4}$$

$$\sqrt{\frac{1}{x^3}} = \frac{1}{x\sqrt{x}} = \frac{1}{x\sqrt{x}} \times \frac{\sqrt{x}}{\sqrt{x}} = \frac{\sqrt{x}}{x(x)} = \frac{\sqrt{x}}{x^2}$$

This is an opportunity to create a graphic organizer, such as the following, to summarize student work with the different concepts related to radicals.



General Outcome: Develop number sense and logical reasoning.

Suggested Assessment Strategies

Performance

- Ask students, in groups of two, to play *Three in a Row* using the operations of radicals. Provide students with a 4×4 game board grid. Players take turns placing a marker in a box on the grid. The player must correctly solve the problem and verify with his/her group that he/she is correct. If the solution is correct, the player places his/her marker in that box. If the solution is incorrect, the other player can steal the box by giving the correct solution. The winner is the player who has three of his/her game markers in a row (vertical, horizontal, or diagonal). A sample grid is shown below:

	1	2	3	4
1	$\left(2\sqrt{3}\right)\left(4\sqrt{5}\right)$	$\dfrac{5x}{\sqrt{5x}}$	$(2\sqrt{x})(3\sqrt{2x})$	$(3-\sqrt{x})^2$
2	$\sqrt{12}(\sqrt{2}+\sqrt{3}-\sqrt{12})$	$\dfrac{\sqrt{24}}{\sqrt{3}}$	$\sqrt{9x}+5\sqrt{16x}$	$\dfrac{\sqrt{27x}+\sqrt{3x}}{\sqrt{3x}}$
3	$\sqrt{50x^3}-5x\sqrt{2x}$	$\dfrac{8\sqrt{8}}{-2\sqrt{2}}$	$\left(6\sqrt{2x}\right)^2$	$\dfrac{8}{\sqrt{2}}-\dfrac{4}{2\sqrt{2}}$
4	Determine the perimeter of a triangle having side lengths $3x\sqrt{2x}$, $\sqrt{32x^3}$ and $x\sqrt{8x}$.	Calculate the area of a rectangle if the dimensions are $5\sqrt{x}+4$ and $3\sqrt{x}$.	$2\sqrt{98}-3\sqrt{50}$	$\left(\sqrt{2}+\sqrt{18}\right)^2$

(NL3.6, NL3.7)

Paper and Pencil

- Einstein's famous formula $E=mc^2$ relates energy E, mass m and the speed of light c. Ask students to express c in terms of E and m and rationalize the denominator.

(NL3.7)

Resources/Notes

Authorized Resource

Principles of Mathematics 11

4.3: Multiplying and Dividing Radicals
SB: pp.191-201
TR: pp.203-209

4.4: Simplifying Algebraic Expressions Involving Radicals
SB: pp.204-213
TR: pp.211-215

Number and Logic

Outcomes

Students will be expected to

NL4 Solve problems that involve radical equations (limited to square roots or cube roots).
[C, PS, R]

Achievement Indicators:

NL4.1 *Determine any restrictions on values for the variable in a radical equation.*

NL4.2 *Determine, algebraically, the roots of a radical equation, and explain the process used to solve the equation.*

NL4.3 *Verify, by substitution, that the values determined in solving a radical equation are roots of the equation.*

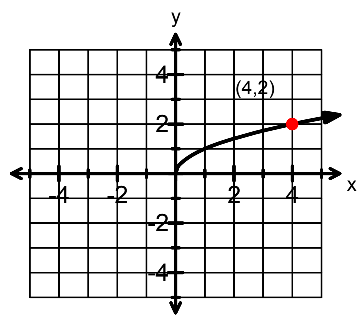
NL4.4 *Explain why some roots determined in solving a radical equation are extraneous.*

Elaborations—Strategies for Learning and Teaching

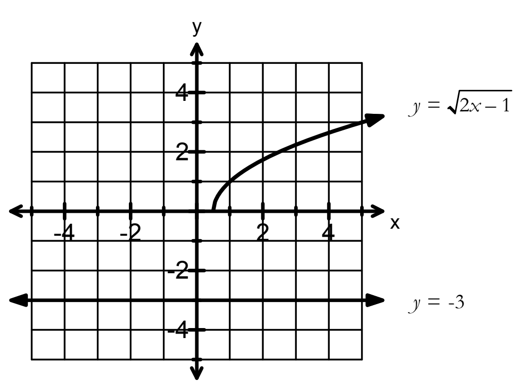
Students will progress from simplifying radical expressions to solving radical equations, limited to a single square root or cube root.

Students will solve radical equations involving a single radical expression containing variables that are first degree. They will be responsible for solving radical equations which result in a linear equation. Students will first work with equations involving square roots and then apply similar strategies to solve equations with cube roots.

As students work with radical equations containing square roots, it would be beneficial to first compare the radical equation to its graph to develop an understanding of restrictions for the variable and the points that satisfy the equation. Using a table of values, ask students to graph $y = \sqrt{x}$.



Focusing on the point (4,2) ask students how they would algebraically solve the equation given only the *y*-coordinate 2. When solving $2 = \sqrt{x}$, the value of *x* can be determined by inspection. Students could also use the idea that squaring a number is the inverse operation of taking the square root. This technique may seem straight forward but students should be exposed to equations where the value is not a solution to the original equation. Consider the example $\sqrt{2x-1} = -3$. The left-hand side of the equation calls for a positive square root, but the right-hand side of the equation is negative. Intuitively, there can be no solution.



General Outcome: Develop number sense and logical reasoning.

Suggested Assessment Strategies

Performance

- Ask students to participate in *Commit and Toss.* Provide students with a selected response problem as shown below. They anonymously commit to an answer and provide a justification for the answer they selected. Students crumble their solutions into a ball and toss the papers into a basket. Once all papers are in the basket, ask students to reach in and take one out. Ask students to then move to the corner of the room designated to match their selected response. In their respective corners, they should discuss the similarities or differences in the explanations provided and report back to the class.

  > What is the value of x in the equation $\sqrt{-3x+6}=6$?
  >
  > (A) -10
  > (B) -2
  > (C) 0
  > (D) 10
  >
  > Explain your reasoning:

  (NL4.1, NL4.2, NL4.3, NL4.4)

- Ask students to participate in a game of *Showdown*. Divide the class into groups of 3 or 4. Each group should select a team captain to set the pace of work. Each student should be given a set of blank answer slips and each group should be given a team answer sheet. The team captain shows problem 1. All team members work individually and silently on the problem. Each student will record their answer on the answer slip turning it over when done. When the captain sees that all of the slips are turned over, he/she calls "Showdown". Group members show and compare their answers, explain their work, and then come up with a group answer. The captain writes the group answer on the team answer sheet. The captain turns the answer card over and compares the given answer to the team's answer. If the answer is different, they discuss the possible errors. Move on to the next problem and repeat. Sample cards are shown below.

Problem 1	Answer 1
Solve: $\sqrt{x-2}=3$	11
Problem 2	**Answer 2**
Simplify: $6\sqrt{7}+2\sqrt{7}-\sqrt{63}$	$5\sqrt{7}$

(NL3.1, NL3.3, NL3.4, NL3.5, NL3.6, NL4.1, NL4.2, NL4.3, NL4.4)

Resources/Notes

Authorized Resource

Principles of Mathematics 11

4.5 Exploring Radical Equations
SB: pp. 214-215
TR: pp.216-219

4.6 Solving Radical Equations
SB: pp. 216-224
TR: pp.220-226

Number and Logic

Outcomes

Students will be expected to

NL4 Continued...

Achievement Indicators:

NL4.1, NL4.2

NL4.3, NL4.4 *Continued*

Elaborations—Strategies for Learning and Teaching

Students should recognize that given the *y*-coordinate of -3, there is no possible *x*-coordinate that satisfies the equation. However, squaring both sides of the equation results in $x = 5$. This cannot be correct, as both substitution and the graph have shown that the equation has no solution. This leads in to the concept of extraneous roots.

It is important for students to recognize that squaring both sides of an equation creates a new equation which may not be equivalent to the original radical equation. Consequently, solving this new equation may create solutions that didn't previously exist. After solving for the variable, reinforce the importance of checking that the value is a solution to the original equation. Any extraneous roots are rejected as answers.

Another strategy students can use to solve a radical equation involves applying a power that will eliminate the radical expression on both sides of the equation. To solve $\sqrt{x+1} = 4$, for example, students would first rewrite the radical expression with a rational exponent, resulting in $(x+1)^{\frac{1}{2}} = 4$. Using the multiplicative inverse gives $\left((x+1)^{\frac{1}{2}}\right)^2 = (4)^2$ and leads to an equation without radicals.

Students should be exposed to equations where the radical is not isolated. In such situations, make a comparison to solving a linear equation. They should recognize that solving an equation such as $3+\sqrt{2x+1} = 7$ follows a process that is similar to solving $3 + x = 7$.

Rather than squaring both sides of a radical equation, students sometimes mistakenly square the individual terms. When solving $3+\sqrt{2x+1} = 7$, for example, they may not isolate the radical. Squaring each term results in the incorrect equation, $3^2 + (\sqrt{2x+1})^2 = 7^2$.

The following illustration could be used to reinforce why squaring individual terms of an equation is not the same as squaring both sides of the equation.

$3 + 4 = 7$	$3 + 4 = 7$
$3^2 + 4^2 = 7^2$	$(3 + 4)^2 = 7^2$
$9 + 16 = 49$	$7^2 = 7^2$
$25 \neq 49$	$49 = 49$

General Outcome: Develop number sense and logical reasoning.

Suggested Assessment Strategies

Performance

- In groups of two, ask students to participate in the activity *Pass the Problem.* Give each pair a problem that involves solving a radical equation. Have one student write the first line of the solution and pass it to the second student. The second student will verify the workings and check for errors. If there is an error present, ask students to discuss the error and why it occurred. The student will then write the second line of the solution and pass it their partner. This process continues until the solution is complete.

(NL4.1, NL4.2, NL4.3)

Resources/Notes

Authorized Resource

Principles of Mathematics 11

4.5 Exploring Radical Equations
SB: pp. 214-215
TR: pp.216-219

4.6 Solving Radical Equations
SB: pp. 216-224
TR: pp.220-226

Number and Logic

Outcomes

Students will be expected to

NL4 Continued...
Achievement Indicators:

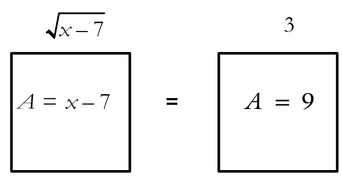
NL4.1, NL4.2
NL4.3, NL4.4 *Continued*

NL4.5 Solve problems by modelling a situation with a radical equation and solving the equation.

Elaborations—Strategies for Learning and Teaching

The area model can also be used to solve radical equations. In Grade 8, students viewed the area of the square as the perfect square number, and either dimension of the square as the square root (8N1). Recall that if a square has an area of 4, then its side has a length of 2. Similarly, if a square has an area of 3, then its side has a length of $\sqrt{3}$. Teachers should prompt discussion about the side length of a square if its area is *x*. Consider the following example: Solve $\sqrt{x-7}=3$.

Using the area model, students will label the dimensions of one square as $\sqrt{x-7}$ and the dimension of the other square as 3. They can then determine the area of each square.



Students should recognize that the goal is to determine the value of *x* which results in the same area for both squares. Solving the equation $x - 7 = 9$ results in $x = 16$. This representation helps students visualize what each equation is describing. Encourage them to check their answers by substituting the value back into the original equation.

Students will apply similar strategies when working with radical equations containing a cube root. They will begin by isolating the radical on one side of the equation and cubing both sides of the equation. Encourage students to continue to verify all solutions to identify extraneous roots. This would be a good opportunity to discuss restrictions when the radical is a square root compared to a cube root. Students must restrict the variable to ensure that the radicand of a square root is positive. The radicand of a cube root, however, may be negative.

Students will be exposed to real-world examples where the equation may contain a radical that is a square root or cube root. They will solve for the unknown variable by either squaring or cubing both sides of the equation. Provide students with the following example and ask them to answer the questions:

- Collision investigators can approximate the initial velocity, *v*, in kilometres per hour, of a car based on the length, *l*, in metres, of the skid mark. The formula $v = 12.6\sqrt{l} + 8, l \geq 0$ models the relationship. What length of skid is expected if a car is travelling 50 km/hr when the brakes are applied? How is knowledge of radical equations used to solve this real world problem?

General Outcome: Develop number sense and logical reasoning.

Suggested Assessment Strategies

Paper and Pencil

- Ask students to state restrictions, solve and check for extraneous roots.
  (i) $\sqrt{4x} = 8$
  (ii) $\sqrt{x+4} = 5$
  (iii) $\sqrt{2x-3} = -2$
  (iv) $\sqrt[3]{x-2} = -3$

(NL4.1, NL4.2, NL4.3, NL4.4)

- Ask students to answer the following:
  (i) The period T (in seconds) is the time it takes a pendulum to make one complete swing back and forth. This is modelled by $T = 2\pi\sqrt{\frac{L}{32}}$, where L is the length of the pendulum in feet. Ask students to determine the period of the pendulum if its length is 2 ft.

  (NL4.5)

  (ii) The radius of a cylinder can be found using the equation $r = \sqrt{\frac{V}{\pi h}}$ where r is the radius, V is the volume, and h is the height. A cylindrical tank can hold 105.62 m^3 of water. If the height of the tank is 2 m, what is the radius of its base?

  (NL4.5)

  (iii) The surface area (S) of a sphere with radius r can be found using the equation $S = 4\pi r^2$.
  (a) Using the given equation, how could you find the radius of a sphere given its surface area? Write the equation.
  (b) The surface area of a ball is 426.2 cm^2. What is its radius?

  (NL4.5)

  (iv) Research applications of radicals in physics, biology or other subject areas. Exchange the problems with other classmates and ask them to solve the problem.

  (NL4.5)

Resources/Notes

Authorized Resource

Principles of Mathematics 11

4.5 Exploring Radical Equations
SB: pp. 214-215
TR: pp.216-219

4.6 Solving Radical Equations
SB: pp. 216-224
TR: pp.220-226

Number and Logic

Outcomes

Students will be expected to

NL2 Analyze puzzles and games that involve spatial reasoning, using problem-solving strategies.

[CN, PS, R, V]

Achievement Indicators:

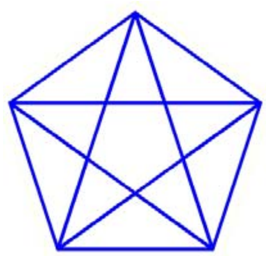
NL2.1 *Determine, explain and verify a strategy to solve a puzzle or to win a game; e.g.,*

- *guess and check*
- *look for a pattern*
- *make a systematic list*
- *draw or model*
- *eliminate possibilities*
- *simplify the original problem*
- *work backward*
- *develop alternative approaches*

NL2.3 Identify and correct errors in a solution to a puzzle or in a strategy for winning a game.

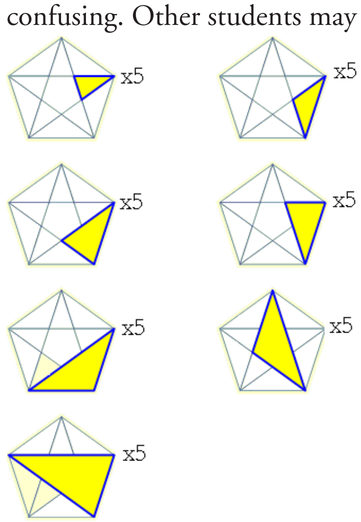
Elaborations—Strategies for Learning and Teaching

There are many strategies that can be used to solve puzzles. The trick is finding the one that works best for the student. Provide students the following example to see if they use patterns to find the total number of triangles in the pentagon below.



http://www.puzzles.com

Some students may shade in or outline the triangles as they find them. They may redraw the puzzle several times so the shading will not get too confusing. Other students may look for a pattern by grouping.



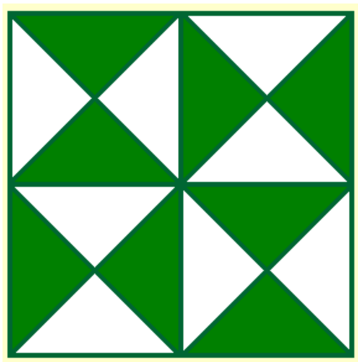
There are seven groups of triangles where each group consists of exactly five triangles with every triangle rotated 72 degrees around the center of the pentagon. So the total number of the triangles in the pentagon is 35.

General Outcome: Develop number sense and logical reasoning.

Suggested Assessment Strategies

Interview

- Ask students to determine the number of triangles that can be found in the following pattern and to explain the strategy they used.



(NL2.1, NL2.3)

Resources/Notes

Authorized Resource

Principles of Mathematics 11

4.3: Multiplying and Dividing Radicals
SB: pp.191-201
TR: pp.203-209

Statistical Reasoning

Suggested Time: 12 Hours

Unit Overview

Focus and Context

In this unit, students will analyse and interepet data using statistical tools such as measures of central tendency, measures of dispersion, normal distribution and z-scores to help them make decisions.

Surveys and polls are frequently used in society. Students will also explore the concepts of confidence interval and margin of error to support a particular position.

Outcomes Framework

GCO

Develop statistical reasoning.

SCO S1

Demonstrate an understanding of normal distribution, including:

- standard deviation
- z-scores.

SCO S2

Interpret statistical data, using:

- confidence intervals
- confidence levels
- margin of error.

Process Standards Key

[C]	Communication	**[PS]**	Problem Solving
[CN]	Connections	**[R]**	Reasoning
[ME]	Mental Mathematics and Estimation	**[T]**	Technology
		[V]	Visualization

SCO Continuum

Mathematics 1201	Mathematics 2201	Mathematics 3201
	Statistics	
	S1 Demonstrate an understanding of normal distribution, including: • standard deviation • z –scores. [CN, PS, T, V]	
	S2 Interpret statistical data, using: • confidence intervals • confidence levels • margin of error. [C, PS, R]	

Statistics

Outcomes

Students will be expected to

S1 Demonstrate an understanding of normal distribution, including:

- standard deviation
- z –scores.

[CN, PS, T, V]

Achievement Indicator:

S1.1 *Use dispersion to describe the differences between two sets of data.*

Elaborations—Strategies for Learning and Teaching

In Grade 7, students explored the measures of central tendency, range and the effect of outliers on a given data set (7SP1). In Grade 8, they examined and created circle graphs, bar graphs, pictographs, line graphs and double bar graphs. They differentiated between graphs that were accurate and graphs that were misleading (8SP1). In Grade 9, both biased and unbiased data collection methods were examined (9SP1).

In this unit, students will work with standard deviation, normal distribution curve and z-scores.

In Grade 7, students described data using the measures of central tendency and range (7SP1). Mean, median and mode were classified as measures of central tendency. When deciding which measure of central tendency best represented a set of data, the effect of outliers were considered.

The range, based on the two extreme values of the data set, is one measure of dispersion. It gives a general idea about the total spread of the data but gives no weight to the central values of the data.

Students should be exposed to situations, such as the following, where the measures of central tendency are not always sufficient to represent or compare sets of data.

Tim and Luke are both enrolled in Mathematics 2201 and scored the following marks on the last five unit tests.

	Unit 1	Unit 2	Unit 3	Unit 4	Unit 5
Tim	60	65	70	75	80
Luke	68	69	70	71	72

When students calculate the measures of central tendency, they should recognize that both sets of data have a mean of 70, median of 70 and no mode. However, there are differences. Ask students about the range values and what information can be learned from these values. Use key terms such as “spread”of the data or “variation” from the mean to prompt discussion. Using the test marks, Tim’s marks (range 20) are more spread out than Luke’s marks (range 4). In other words, Luke’s marks are more clustered around the mean than Tim’s marks.

Range is commonly used as a preliminary indicator of dispersion. However, because it takes into account only the scores that lie at the two extremes, it is of limited use. Later in this unit, a more complete measure of dispersion known as standard deviation will be introduced. It takes into account every score in a distribution.

General Outcome: Develop statistical reasoning.

Suggested Assessment Strategies

Paper and Pencil

- In a science experiment, students tested whether compost helped plants grow faster by counting the number of leaves on each plant. The following results were obtained:

Plant Growth without Compost (# of leaves per plant)	Plant Growth with Compost (# of leaves per plant)
6	6
4	11
5	1
4	6
8	2
3	4

Ask students to answer the following:

(i) Calculate the mean, median and mode for each group.
(ii) Calculate the range for each group.
(iii) Describe the dispersion in the data for each group.
(iv) Which group of plants grew better? Justify your decision.

(S1.1)

- Ask students to calculate the range of each group. They should explain why the range, by itself, can be a misleading measure of dispersion.

Group A: 8, 13, 13, 14, 14, 14, 15, 15, 20
Group B: 7, 7, 8, 9, 11, 13, 15, 15, 17, 18

(S1.1)

Resources/Notes

Authorized Resource

Principles of Mathematics 11

5.1: Exploring Data

Student Book(SB): pp.238-240
Teacher Resource(TR): pp.251-254

Statistics

Outcomes

Students will be expected to

S1 Continued...
Achievement Indicator:

S1.2 *Create frequency tables and graphs from a set of data.*

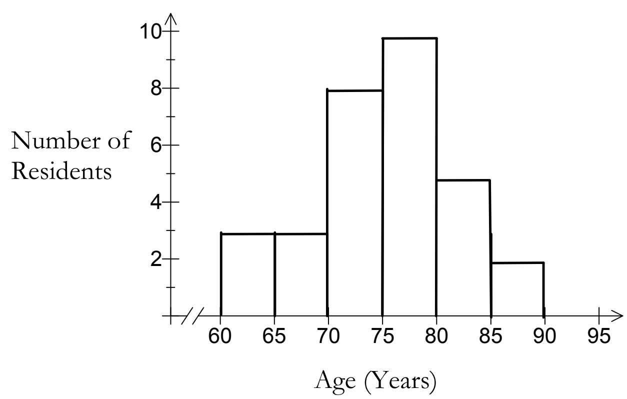
Elaborations—Strategies for Learning and Teaching

Introduce frequency distribution as a set of intervals that can be dispalyed in three forms: a table, a histogram and a frequency polygon. Discussion around an appropriate number of intervals and the interval width (i.e., bin width) is important here.

Histograms and frequency polygons provide a pictorial representation of the data. This provides an opportunity for students to draw conclusions and make inferences from the data. Frequency polygons are especially helpful when comparing multiple sets of data because they can be graphed on top of each other.

Unlike a bar graph, students should recognize there are no gaps between the bars in a histogram. Discuss this as it relates to continuous and discrete data. Each bar is constructed using the lower and upper limits of the interval. Consider the following graph:

Age of Residents at SunnyView Seniors Home



The first bar in this graph begins at 60 on the horizontal axis and ends at 65. The second bar starts where the first bar ends, namely 65-70. As the intervals are required to be non-overlapping, the convention is that the lower limit of each interval includes the number. For example, in the above graph, the data value 65 would be placed in the 65 - 70 interval.

To plot each vertex for the frequency polygon, students can determine the midpoint of the interval aand then join the vertices with line segments. Ensure students connect the endpoints to the horizontal axis.

Students exposure to histograms and frequency polygons will help them when they investigate the properties of a normal distribution later in this unit.

General Outcome: Develop statistical reasoning.

Suggested Assessment Strategies

Paper and Pencil

- The marks awarded for an assignment for a Grade 11 class of 20 students are given below. Ask students to present this information in a frequency table and display the data on a histogram. They should describe how the data is distributed.

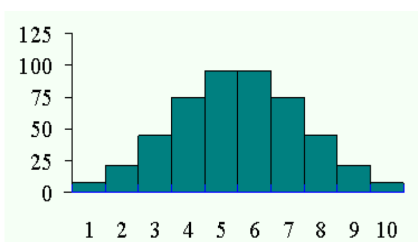
6	7	5	7	7	8	7	6	9	7
4	10	6	8	8	9	5	6	4	8

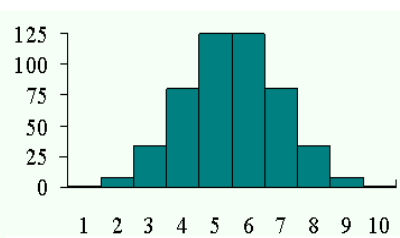
(S1.2)

- Choose an attribute common to all students (number of siblings, number of weekly text messages sent or received, numbers of hours watching television, etc). Ask students to collect the data from the class, select an appropriate bin size and display the information in a frequency table. They should construct a histogram of the data abd then describe the data distribution.

(S1.2)

- Ask students which of the distribution of scores has the larger dispersion? They should justify their reasoning.





(S1.2)

Resources/Notes

Authorized Resource

Principles of Mathematics 11

5.2: Frequency Tables, Histograms, and Frequency Polygons

SB: pp.241-253
TR: pp.255-260

Statistics

Outcomes

Students will be expected to

S1 Continued...
Achievement Indicators:

S1.3 *Explain, using examples, the meaning of standard deviation.*

S1.4 *Calculate, using technology, the population standard deviation of a data set.*

S1.5 *Solve a contextual problem that involves the interpretation of standard deviation.*

Elaborations—Strategies for Learning and Teaching

Students were introduced earlier to range as a measure of dispersion. Remind students the range is only a measure of how spread out the extreme values are, so it does not provide any information about the variation within the data values themselves. Another measure of dispersion is standard deviation. It is useful for comparing two or more sets of data. Consider an example similar to the one studied earlier regarding Mathematics 2201 unit test marks.

	Unit 1	Unit 2	Unit 3	Unit 4	Unit 5
Tim	60	65	70	75	80
Mary	60	69	70	71	80

The measures of central tendency and range for these two data sets are equal yet, clearly there are differences in the two sets of marks. Students determine Tim's marks have a standard deviation of 7.1 and a mean of 70. Mary's marks have a standard deviation of 6.4. Introduce the concept of standard deviation as measuring how far the data values lie from the mean. Using the test marks as an example, prompt discussion around standard deviation by asking questions such as:

- Whose marks are more dispersed? What does this mean in terms of a high or low standard deviation?
- If the data is clustered around the mean, what does this mean about the value of the standard deviation?
- Who was more consistent over the five unit tests?

Students should make the connection that if the data is more clustered around the mean, there is less variation in the data and the standard deviation is lower. If the data is more spread out over a large range of values, there is greater variation and the standard deviation would be higher.

In this course, students will be calculating standard deviation of a population, represented by the symbol σ.

$$\sigma = \sqrt{\frac{\sum\left(x-\bar{x}\right)^2}{n}}$$

Students should be exposed to calculating standard deviation manually to provide an understanding of how standard deviation is determined. This should be done, however, using an example with a small amount of data to keep calculations simple. The benefit of using technology to solve contextual problems is because the data is often large.

General Outcome: Develop statistical reasoning.

Suggested Assessment Strategies

Paper and Pencil

- Ask students to answer the following questions:

  (i) Is it possible for a data set to have a standard deviation of 0? Can the standard deviation ever be negative? Explain why or why not.

  (S1.3)

  (ii) If 5 were added to each number in a set of data, what effect would it have on the mean? On the standard deviation? What if each element in the data set was multiplied by -3?

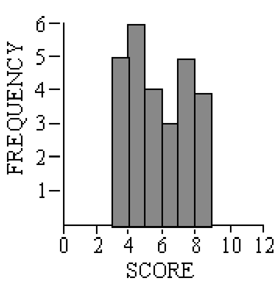
  (S1.3)

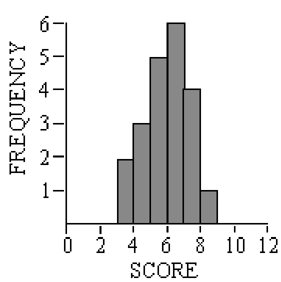
Observation

- Students can work in groups for this activity. Each group is presented with a pair of graphs where the mean for each graph is given. Ask students to discuss the following for each pair of graphs and share their answers with the rest of the class:

  (i) Indicate whether one of the graphs has a larger standard deviation than the other or if the two graphs have the same standard deviation.What approaches did you use to decide which graph had the larger standard deviation?

  (ii) Identify the characteristics of the graphs that make the standard deviation larger or smaller. What features of a histogram seem to have no bearing on the standard deviation? What features do appear to affect the standard deviation?





(S1.3)

Resources/Notes

Authorized Resource

Principles of Mathematics 11

5.3: Standard Deviation

SB: pp.254-268

TR: pp.261-267

Statistics

Outcomes

Students will be expected to

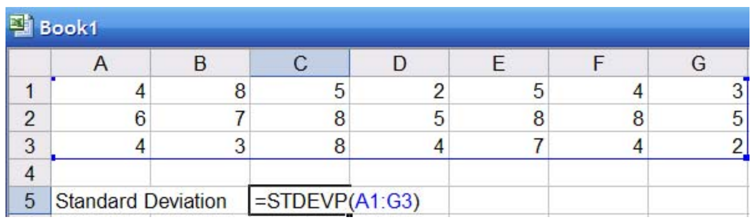
S1 Continued...
Achievement Indicators:

S1.4, S1.5 *Continued*

Elaborations—Strategies for Learning and Teaching

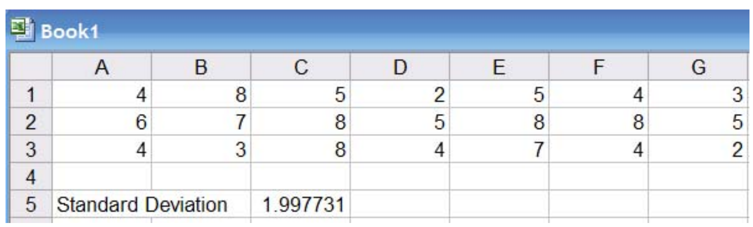
Students are expected to calculate standard deviation using technology such as scientific calculators, graphing calculators or computer software programs (e.g., Microsoft Excel).

When using Microsoft Excel, for example, students input the data and use the notation STDEVP to calculate the population standard deviation. Using the following snapshot, A1:G3 are the cells that contain the data.



Book1

	A	B	C	D	E	F	G
1	4	8	5	2	5	4	3
2	6	7	8	5	8	8	5
3	4	3	8	4	7	4	2
4							
5	Standard Deviation		=STDEVP(A1:G3)				



Book1

	A	B	C	D	E	F	G
1	4	8	5	2	5	4	3
2	6	7	8	5	8	8	5
3	4	3	8	4	7	4	2
4							
5	Standard Deviation		1.997731				

Students should examine the meaning of standard deviation through relevant examples. Promote discussion around the following:

- Sports Illustrated is doing a story on the variation of player heights on NBA basketball teams. The heights, in cm, of the players on the starting line ups for two basketball teams are given in the table below.

Lakers	195	195	210	182	205
Celtics	193	208	195	182	180

The team with the most variation in height will be selected for the cover of Sports Illustrated magazine. Ask students which team will appear on the cover.

General Outcome: Develop statistical reasoning.

Suggested Assessment Strategies

Paper and Pencil

- Two obedience schools for dogs monitor the number of trials required for twenty puppies to learn the *sit and stay* command.

True Companion Dog School	
Number of Trials	Number of Puppies
7	1
8	2
9	5
10	4
11	4
12	4

Dog Top School	
Number of Trials	Number of Puppies
7	4
8	3
9	2
10	3
11	4
12	4

(i) Ask students to determine the mean and standard deviation of the number of trials required to learn the *sit and stay* command.
(ii) Ask students which school is more consistent at teaching puppies to learn the *sit and stay* command.They should justify their reasoning.

(S1.4, S1.5)

Performance

- One of Leonardo da Vinci's well known illustrations shows the proportions of a human figure by enclosing it in a circle. It appears that the arm span of an adolescent is equal to the height.

(i) In groups of two, ask students to measure each others arm span and height. They should collect the data from all students and create two group of data sets.
(ii) Ask students to calculate the mean and standard deviation for each of the two sets of data. They should compare the values and explain what they notice.
(iii) Ask students to calculate the difference between each person's height and arm span. From the data, what would they expect the mean and standard deviation of this data set to be? They should check their prediction for ten students in the class.

(S1.4, S1.5)

Resources/Notes

Authorized Resource

Principles of Mathematics 11

5.3: Standard Deviation

SB: pp.254-268

TR: pp.261-267

Web Links

A standard deviation calculator showing all the steps in calculating the population standard deviation can be found at the following website:

http://www.mathsisfun.com/data/standard-deviation-calculator.html

The following website calculates the mean and population standard deviation without showing any calculations:

http://www.alcula.com/calculators/statistics/standard-deviation/

Statistics

Outcomes

Students will be expected to

S1 Continued...

Achievement Indicator:

S1.6 *Explain, using examples, the properties of a normal curve, including the mean, median, mode, standard deviation, symmetry and area under the curve.*

Elaborations—Strategies for Learning and Teaching

In many situations, if a controlled experiment is repeated and the data is collected to create a histogram, the histogram becomes more and more bell-shaped as the number of data in the distribution increases. Such repeated measurements will produce bell-shaped distributions which exhibit the characteristics of a normal curve.

Introduce students to the shape of the normal curve using an interactive plinko game (*www.math.psu.edu/dlittle/java/probability/plinko/index.html* or *www.mathsisfun.com/data/quincunx.html)*. The general shape of the histogram will begin to approach a normal curve as more trials are added. They should observe the following characteristics:

- Normal distributions are symmetrical with a single peak at the mean of the data.
- The curve is bell shaped with the graph falling off evenly on either side of the mean.

Graphing a normal distribution by hand can be tedious and time consuming. Using techonlogy, such as graphing calculators and Microsoft Excel, students can investigate the properties of a normal distribution and work with data that is specifically one, two and three standard deviations from the mean. Consider the following data sample:

Number of hours playing video games in a week (15year old students)									
21.1	21.8	17.7	23.4	14.9	20.8	19.1	18.6	25.6	23.3
18.5	19.1	18.8	18.3	20.2	15.5	21.4	16.7	14.7	20.5
17.9	20.9	24.5	22.2	17.1	24.5	21.1	18.1	21.0	16.2
23.7	18.9	21.3	16.0	20.2	27.3	17.6	20.7	19.7	14.9

Guide students through the problem asking them to answer the following:

- Determine the mean, standard deviation, median and mode. They should explain what they notice about the values.
- Construct a frequency table to generate a histogram. Use an interval width equal to the standard deviation since the spread of the normal distribution is controlled by the standard deviation.
- Discuss the symmetry of the histogram.
- Draw a frequency polygon and explain its shape.

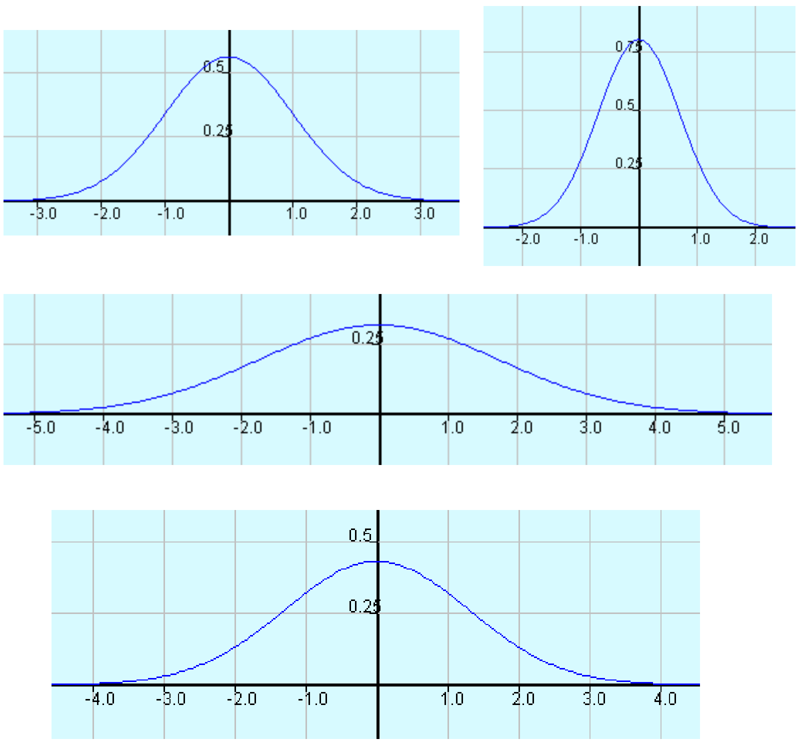
Students should realize in a normal distribution the data is symmetrical about the mean. Since the mean lies in the middle of the data it is also the median. As the mean is located where the curve is at its highest point, it is also the mode. Therefore the mean, median and mode are equal for a normal distribution.

General Outcome: Develop statistical reasoning.

Suggested Assessment Strategies

Journal

- Ask students to respond to the following prompt:
  Which normal distribution curve has the largest standard deviation? Explain your reasoning.



(S1.6)

Resources/Notes

Authorized Resource

Principles of Mathematics 11

5.4: Normal Distribution

SB: pp.269-282
TR: pp.268-275

Web Links

An interactive dice game which demonstrates the histogram as you input the number of rolls.
http://nces.ed.gov/nceskids keyword:chances

An interactive plinko game demonstrating the histogram can be found at the following site:
http://phet.colorado.edu/sims/plinko-probability/plinko-probability_en.html

The following website gives step by step instructions on how to draw a normal distribution curve and create a histogram in excel.
http://www.statisticshowto.com/articles/category/microsoft-excel/

Statistics

Outcomes

Students will be expected to

S1 Continued...
Achievement Indicators:

S1.6 *Continued*

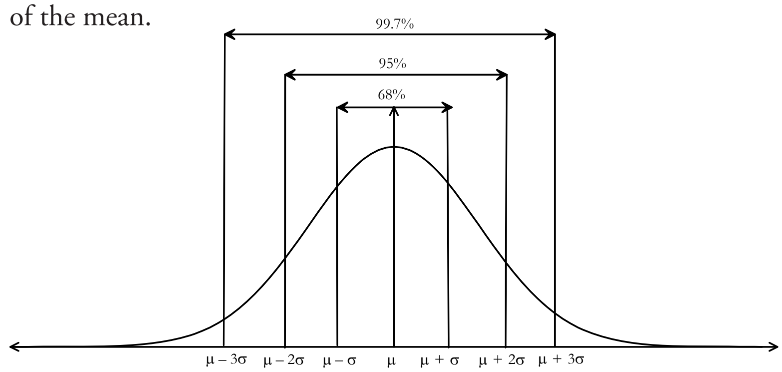
S1.7 *Determine if a data set approximates a normal distribution and explain the reasoning.*

Elaborations—Strategies for Learning and Teaching

Using the previous data, continue to investigate the area under the curve and the percent of data that is one, two, and three standard deviations from the mean. Coach students along the investigation using the following directions and discuss their observations.

- Approximate the percentage of data that is to the left of the mean and then to the right of the mean. Approximate the percentage of data within three standard deviations. Ask them what they notice.
- Approximate the percentage of data within one standard deviation. Approximate the percentage of data within two standard deviations.

As students analyze the data, they should recognize that fifty percent of the distribution lies to the left of the mean and fifty percent lies to the right of the mean. Furthermore, approximately 68% of the data lies within one standard deviation of the mean, 95% is within two standard deviations of the mean, and 99.7% is within three standard deviations of the mean.



Students will have to confirm whether a data set approximates a normal distribution. Observe students as they are working through examples to make sure they are understanding the concept of normal distribution. With the aid of technology, it is important for students to consider the the measures of central tendency, the appearance of the histogram and the percent of data within one, two and three standard deviations of the mean (i.e, are they close to 68%, 95% and 99.7%).

If the data approximates a normal distribution, the properties of the normal distribution can be used to solve problems. Students should recognize that for normally distributed data, the area under the curve between any two values is equal to the probability that a given data value will fall between those two values. Consider a data set of 100 items normally distributed having a mean of 3.4 and a standard deviation of 0.2. If 68% of the area under the curve is within one standard deviation, then students should realize approximately 68 items should be between 3.2 and 3.6.

General Outcome: Develop statistical reasoning.

Suggested Assessment Strategies

Observation

- There are a variety of Java applets available online to manipulate the parameters (mean and standard deviation) to show the characteristics and how they affect the shape of the curve. Ask students to investigate these characteristics and answer the questions:
  - (i) How are the standard deviation and the shape of the graph related?
  - (ii) Does the area under the curve change as the number of standard deviations changes?
  - (iii) Does the area under the curve change as the value of the standard deviation changes?

(S1.6)

Paper and Pencil

- A data set of 50 items is given below with a standard deviation of 1.8. Ask students to answer the following questions:

6	9	6	7	6	7	6	7	6	6
5	8	5	8	6	7	7	4	8	6
5	4	6	8	6	6	7	6	6	8
8	7	6	6	0	4	7	8	5	3
6	7	6	7	1	7	7	4	4	3

  - (i) What is the value of the mean, median and mode?
  - (ii) Is the data normally distributed? Explain your reasoning.

(S1.7)

Resources/Notes

Authorized Resource

Principles of Mathematics 11

5.4: Normal Distribution

SB: pp.269-282
TR: pp.268-275

Web Link

An interactive graph of the normal curve can be found at: *http://www.measuringusability.com/normal_curve.php*. This site is beneficial to students when answering questions under the *Observation* Assessment Strategy.

Statistics

Outcomes

Students will be expected to

S1 Continued...
Achievement Indicators:

S1.8 *Compare the properties of two or more normally distributed data sets.*

S1.9 *Solve a contextual problem that involves normal distribution.*

S1.10 *Determine, with or without technology, and explain the z-score for a given value in a normally distributed data set.*

Elaborations—Strategies for Learning and Teaching

Comparisons of normal distributions involve comparing the mean and standard deviation of each. The mean will determine the location of the centre of the curve on the horizontal axis. The standard deviation will determine the width and height of the curve. Assume two sets of data have the same mean, for example, but different standard deviations. The distribution of data with a lower standard deviation would result in a graph that is taller and narrower.

Students will use the properties of the normal curve to solve problems. Encourage students to sketch the normal distribution curve to help them visualize the information. Consider the following example.

- The assessed value of housing in Bay Roberts is normally distributed. There are 1200 houses with a mean assessed value of $180 000. The standard deviation is $10 000. What percentage of houses have an assessed value between $170 000 and $200 000? How many houses have an assessed valued between $170 000 and $200 000?

The example above implies that students will not only be required to calculate the percentage of data items within an interval, but using the total number of data items, they will calculate the actual number within the interval.

Discuss with students the importance of data analysis and its implications in the real world. Consider the following scenerios:

- We may want to talk about particular scores within a set of data.
- We may want to tell other people about whether or not a score is above or below average.
- We may want to indicate how far away a particular score is from the average.
- We might also want to compare scores from different sets of data and figure out which score is better.

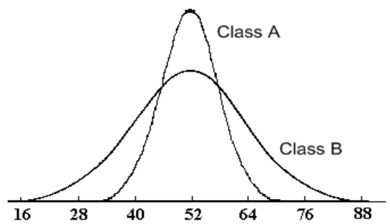
Z-scores can help with all of these scenerios. It is a standardized value that indicates the number of standard deviations a data value is from the mean. If a data value is 2 standard deviations left of the mean, for example, then $z = -2$. When students convert normally distributed scores into z-scores, they can determine the probabilities of obtaining specific ranges of scores using a z-score table. This table is based on a normal distribution that has a mean of 0 and a standard deviation of 1. Students are no longer restricted to data values that are exactly one, two or three standard deviations from the mean. They will now use z-scores to determine the percent of data that is also less than or greater than any particular data value or even between two data values.

General Outcome: Develop statistical reasoning.

Suggested Assessment Strategies

Paper and Pencil

- The graph below shows the scores on a standardized test, normally distributed, for two classes.



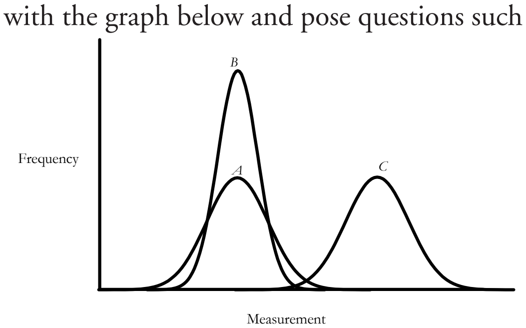
Ask students to answer the following questions:

(i) What do these graphs have in common? How are they different?
(ii) Which graph has the greatest standard deviation? Why?
(iii) If a student scored 42 on a test, which class are they most likely in? Explain your answer. (S1.8)

- Sally has a height of 1.75 m and lives in a city where the average height is 1.60 m and the standard deviation is 0.20 m. Leah is 1.80 m and lives in a city where the average height is 1.70 m and the standard deviation is 0.15 m. Ask students to identify which of the two is considered to be taller compared to their fellow citizens. They should explain their reasoning. (S1.9)

Interview

- Present students with the graph below and pose questions such as the following:



(i) What do graphs A and B have in common? How are they different?
(ii) What do graphs A and C have in common? How are they different?
(iii) Is there any relationship between graphs B and C?
(iv) Which graph has the smallest standard deviation? Why?
(v) Why is the graph with the lowest standard deviation also the tallest?
(vi) Which graph has the greatest mean? How do you know? (S1.8)

Resources/Notes

Authorized Resource

Principles of Mathematics 11

5.4: Normal Distribution

SB: pp.269-282
TR: pp.268-275

5.5: Z-Scores

SB: pp.283-294
TR: pp.276-281

Statistics

Outcomes

Students will be expected to

S1 Continued...
Achievement Indicator:

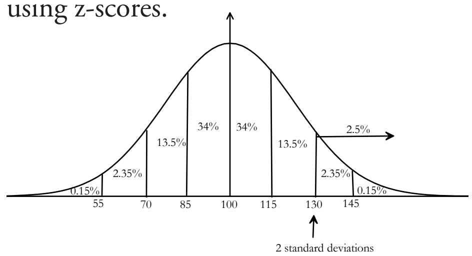
S1.10 *Continued*

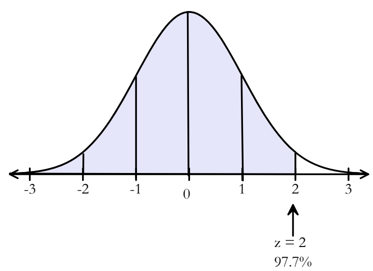
Elaborations—Strategies for Learning and Teaching

Introduce students to the z-score formula $z = \frac{x-\mu}{\sigma}$.
Students can work with either normal distriution or z-scores when trying to find the percent of data that is one, two, and three standard deviations from the mean. Consider the following example:

- IQ tests are normally distributed with a mean of 100 and a standard deviation of 15. What percentage of students achieved less than the 130 mark?

Ask students to first draw the normal distribution curve, labeling the mean and standard deviation. They should then verify their answer using z-scores.

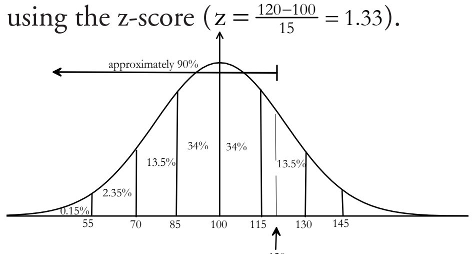


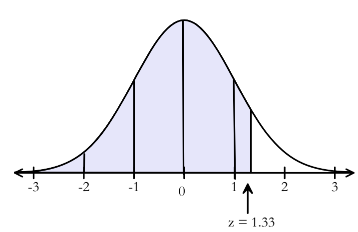


Using a visual representation, students should see that 97.5% of the people would receive a mark less than 130. Since the score of 130 is 2 standard deviations away from the mean, students can use the z-score table to find the area under the curve to the left of the standard normal curve. This value is 0.9772 which is 97.7%. Discuss with students why there is a small error in the calculation when compared to standard deviation.

Expose students to examples where they can use z-scores to determine the percentage of data that is less than or greater than a particular data value, not limited to one, two or three standard deviations from the mean.

Using the previous example, ask students to consider a data value that is between 1 and 2 standard deviations from the mean such as a score of 120. Ask them to determine the percentage of students who achieved less than the 120 mark? They should first estimate their answer (using standard deviation and the normal distribution curve) and then verify it using the z-score ($z = \frac{120-100}{15} = 1.33$).





General Outcome: Develop statistical reasoning.

Suggested Assessment Strategies

Paper and Pencil

- Ask students to answer the following questions:
    (i) The average life expentancy of a certain breed of cat was determined to be 12.2 years with a standard deviation of 1.3 years. What is the probability that a given cat wil live less than 14 years?

    (S1.10)

    (ii) On the math placement test at Memorial University of Newfoundland, the mean score was 62 and the standard deviation was 11. If Mark's z-score was 0.8, what was his actual exam mark?

    (S1.10)

Resources/Notes

Authorized Resource

Principles of Mathematics 11

5.5: Z-Scores

SB: pp.283-294
TR: pp.276-281

Statistics

Outcomes

Students will be expected to

S1 Continued...
Achievement Indicator:

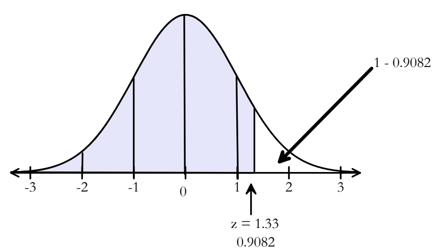
S1.10 *Continued*

Elaborations—Strategies for Learning and Teaching

Engage students in a discussion by asking the following questions:

- Was your estimate reasonable when you compared it the the z-score?
- Why is the z-score more reliable than estimating using standard deviation?
- What percentage of students achieved less than the 120 mark?

Expose students to examples where they also have to determine the area under the curve to the right of the z-score. They should realize the z-score table can still be used in a similar manner to determine the area under the curve. If students were asked to determine the percentage of students who achieved greater than the 120 mark, they would write 1 - 0.9082 since the area under the curve is 1.



The z-score can also provide a standard measure for comparing two different normal curves by transferring each to the standard normal distribution curve, having a mean of zero and a standard deviation of one. Consider the following example:

- On her first math test, Susan scored 70%. The mean class score was 65% with a standard deviation of 4%. On her second test she received 76%. The mean class score was 73% with a standard deviation of 10%. On which test did Susan perform better with respect to the rest of her class?

If students examine Susan's test scores alone they would probably conclude she performed better on the second unit test. However, this is not necessarily the case. We must take into account the overall performance of the class to compensate for other factors such as the difficulty of the material tested. By calculating Susan's z-score values for each test we can compare Susan's performance on both tests to the same standard.

On her Unit 1 test, Susan's z-score was $z = \frac{70-65}{4} = 1.25$, which implies Susan scored 1.25 standard deviations above the mean. On her Unit 2 test, Susan's z-score was $z = \frac{76-73}{10} = 0.3$. On Unit 2, Susan scored 0.3 standard deviations above the mean. Since both z-scores relate to a standard normal distribution, students can compare the values and should conclude that Susan performed better on her Unit 1 exam.

General Outcome: Develop statistical reasoning.

Suggested Assessment Strategies

Paper and Pencil

- Ask students to participate in the following dinner menu activity. Questions in the Appetizer and Dessert section are mandatory. They have a choice in the Entree and Side Dishes category.

Dinner Menu—Z-Scores

Appetizer (Everyone Shares)
What is a z-score? How do you determine it?

Entrée (Select One)
• Battery Inc. claims that its batteries have a mean life of 50 h, with a standard deviation of 6 h. What is the probability, to two decimal places, that a battery will last 57 h or longer? • The speeds of cars on a highway have a mean of 80 km/h with a standard deviation of 8 km/h. What percent of the cars averaged more than 96 km/h?

Side Dishes (Select at Least Two)
• A line up for the tickets to a concert has an average waiting time of 400 s with a standard deviation of 32 s. Sketch a normal distribution curve which illustrates the probability that you wait in line for more than 6 minutes. • What can you say about a data value if you know that its z-score is negative? Zero? • The average playing time of a defense player in the NHL is 17.2 minutes per game with a standard deviation of 2.2 minutes. What is the probability that a defense player will play between 15.6 and 19.2 minutes?

Dessert (Required)
Create and solve a question using a real world example which uses z- scores.

(S1.9, S1.10)

Resources/Notes

Authorized Resource

Principles of Mathematics 11

5.5: Z-Scores

SB: pp.283-294
TR: pp.276-281

Statistics

Outcomes

Students will be expected to

S1 Continued...

Achievement Indicator:

S1.11 *Explain, using examples representing multiple perspectives, the application of standard deviation for making decisions in situations such as warranties, insurance or opinion polls.*

Elaborations—Strategies for Learning and Teaching

Standard deviation is used in society to influence decisions such as length of warranties provided by a company, cost of insurance, quality control, and opinion polls. Since z-scores are affected by standard deviation, they will also play a role in such situations. Consider the following example using two different automobiles, a Chevrolet Corvette and a Honda Civic.

- It is known that the mean value of repairs, due to an accident, for both cars is \$3500. The standard deviation for the Corvette is \$1200, while the standard deviation for the Civic is only \$800. If the cost of repairs is normally distributed, determine the probability that the repairs costs will be over \$5000 for both cars.

Ask students to calculate the z-scores for each:

Corvette: $z = \frac{5000-3500}{1200} = 1.25$; Honda: $z = \frac{5000-3500}{750} = 1.5$

Using the z-score table, the probability that the repairs will cost over \$5000 for the Corvette is 10.56% while the probability that the repairs for the Civic will exceed \$5000 is 6.06%. Students should recognize that the Corvette is nearly twice as likely to have damages exceeding \$5000 than the Civic. Therefore, its insurance premiums will be adjusted accordingly.

General Outcome: Develop statistical reasoning.

Suggested Assessment Strategies

Paper and Pencil

- Cars are undercoated as a protection against rust. A car dealer determines the mean life of protection is 65 months and the standard deviation is 4.5 months. Ask students to answer the following questions:
  (i) What guarantee should the dealer give so that fewer than 15% of the customers will return their cars?
  (ii) The dealer creates a fund, based on the guarantee, from which refunds and repairs are made. It is estimated that about 2500 cars will be undercoated annually. The average repair on returned cars is about $165. How much money should be placed in the fund to cover customer returns?
  (iii) What is the probability that an undercoated car, chosen at random, will be returned in 5 years?

(S1.11)

Resources/Notes

Authorized Resource

Principles of Mathematics 11

5.5: Z-Scores

SB: pp.283-294
TR: pp.276-281

Statistics

Outcomes

Students will be expected to

S2 Interpret statistical data, using:

- confidence intervals
- confidence levels
- margin of error.

[C, CN, R]

Achievement Indicator:

S2.1 *Explain, using examples, the significance of a confidence interval, margin of error or confidence level.*

Elaborations—Strategies for Learning and Teaching

It is intended that the focus of this outcome be on the interpretation of data rather than on mathematical calculations. An entire population is often difficult to study, so we often use a representative population called a sample. If the sample is truly representative, then the statistics generated from the sample will be the same as the information gathered from the population as a whole. It is unlikely, however, that a truly representative sample will be selected. We need to make predictions of how confident we can be that the statistics from our sample are representative of the entire population.

Surveys are used to draw conclusions from a sample. How well the sample represents the larger population depends on two important statistics, the margin of error and confidence level. This can be illustrated with an example such as the following:

A Rent-A Car-company surveys customers and finds that 50 percent of the respondents say its customer service is "very good." The confidence level is cited as 95 percent and the margin of error is ± 3 percent. This information indicates that if the survey were conducted 100 times, then 95 times out of 100, the percent of people who say service is "very good" will range between 47% to 53%.

Students are only required to calculate the confidence interval given the mean and margin of error. They can also determine the mean and margin of error from the confidence interval. Consider the following examples:

- A brand of battery has a mean life expectancy of 12.6 hours with a margin of error of 0.7 hours. As 12.6 – 0.7 = 11.9 and 12.6 + 0.7 = 13.3, the confidence interval would indicate that this particular brand of battery would last between 11.9 hours and 13.3 hours.
- The expiration time of milk is said to occur between 13.5 and 14.7 days. The mean expiration time would be the midpoint of the interval, $\frac{13.5+14.7}{2} = 14.1$. This will allow the calculation of the margin of error 14.7 - 14.1 = 0.6 and 13.5 - 14.1 = -0.6. The margin of error would be ±0.6.

General Outcome: Develop statistical reasoning.

Suggested Assessment Strategies

Paper and Pencil

- A botanist collects a sample of 50 iris petals and measures the length of each. It is found that the mean is 5.55 cm and the standard deviation is 0.57 cm. He then reports that he is 95% confident the average petal length is between 5.39 cm and 5.71 cm. Ask students to answer the following:
  (i) Identify the margin of error, the confidence interval, and the confidence level.
  (ii) Explain what information the confidence interval gives about the population of iris petal length.
  (iii) How would the length of a 99% confidence interval be different from that of a 95% confidence interval?
  (iv) If you did not know the margin of error but you know that the confidence interval is between 5.39 cm and 5.71 cm, how could you determine the margin of error?

  (S2.1)

- A report claims that the average family income in a large city is \$32 000. It states the results are accurate 19 times out of 20 and have a margin of error of ±2500.
  (i) What is the confidence level in this situation? Explain what it means.
  (ii) Explain the meaning of a margin of error of ±2500.

  (S2.1)

Journal

- Ask students to explain what is meant by a 90%, a 95%, and a 99% confidence interval. How are these intervals similar? How are they different?

  (S2.1)

Resources/Notes

Authorized Resource

Principles of Mathematics 11

5.6: Confidence Intervals

SB: pp.295-304
TR: pp.282-288

Statistics

Outcomes

Students will be expected to

S2 Continued...

Achievement Indicator:

S2.2 *Explain, using examples, how confidence levels, margin of error and confidence intervals may vary depending on the size of the random sample.*

Elaborations—Strategies for Learning and Teaching

The sample size of a random sample will affect the confidence interval and margin of error. Conceptualize this concept using an example. The town of Conception Bay South is trying to determine the location of a new recreation centre. Ask students to make a prediction about the confidence interval and margin or error if 100 people were surveyed, if 1000 perople were surveyed or if all the people in the town were surveyed. If 100 people were surveyed, they should realize the results could be skewed one way or the other. In other words, there is no guarantee that the results would be accurate. This would result in a significant margin of error (small sample size) and it would produce a confidence interval with a large range. If 1000 people were surveyed, there is a greater chance for more accurate results. The increase in sample size would decrease the margin of error and, in turn, decrease the range on the confidence interval, thus, getting us closer to the actual result. If all the people in the town were surveyed, students should recognize they would know the actual result and have no margin of error.

General Outcome: Develop statistical reasoning.

Suggested Assessment Strategies

Paper and Pencil

- In a national survey of 400 Canadians from the ages of 20 to 35, 37.5% of those interviewed claimed they exercise for at least four hours a week. The results were considered accurate within 4%, 9 times out of 10. Ask students to answer the following questions:
  - (i) Are you dealing with a 90%, 95%, or 99% confidence interval? How do you know?
  - (ii) How many people in the survey claimed to exercise at least four hours a week?
  - (iii) What is the margin of error?
  - (iv) What is the confidence interval? Explain its meaning.
  - (v) What are some limitations of this survey?
  - (vi) If the writers of the article created a 99% confidence interval based on this data, how would it be different? How would it be the same?
  - (vii) How would the confidence interval change if the sample size was increased to 1000 but the sample proportion remained the same?

(S2.2)

Journal

- In a recent survey, 355 respondents from a random sample of 500 first-year university students claim to be attending their first choice of university. If the sample size was decreased but the sample proportion remained the same, ask students to explain how the confidence intervals would change.

(S2.2)

Resources/Notes

Authorized Resource

Principles of Mathematics 11

5.6: Confidence Intervals

SB: pp.295-304
TR: pp.282-288

Statistics

Outcomes

Students will be expected to

S2 Continued...

Achievement Indicators:

S2.3 *Make inferences about a population from sample data, using confidence intervals, and explain the reasoning.*

S2.4 *Provide examples from print or electronic media in which confidence intervals and confidence levels are used to support a particular position.*

S2.5 *Interpret and explain confidence intervals and margin of error, using examples found in print or electronic media.*

S2.6 *Support a position by analyzing statistical data presented in the media.*

Elaborations—Strategies for Learning and Teaching

Provide students with a mean value and a margin of error or the confidence interval and ask them analyze the data to make inferences. Consider the following example:

- A recent study reports that 61% of students at Lewisporte Intermediate own a cell phone. The results of the study are reported to be accurate, 19 times out of 20, with a margin of errror of 3.6 percent.

Ask students the the following questions:

(i) What is the confidence level?

(ii) What is the confidence interval?

(iii) According to the study, if there are 258 students at Lewisporte Intermediate School, what is the range of students who could own a cell phone?

Provide students with an inference based on given confidence intervals and ask them to comment on the validity of the inference. Consider the following example:

- The city of St. John's is trying to determine whether or not to continue the curb side recycling program. A survey indicted that 50% of residents wanted the program to continue. The survey was reported to be accurate, 9 times out of 10, with a margin of error of 16.7%. Based on these results, what course of action should the city take with respect to the curb side recycling program? Explain your answer.

Students need to realize that the confidence level is only 90% and due to the large margin of error, the range of the confidence interval is also large (33.3% to 66.7%). The city is only 90% confident that between one-third and two-thirds of the population are in favour of the recycling program. Ask students if this is useful information. It should be suggested that the city increase its sample size to reduce the margin of error and use at least a 95% confidence interval to get a better picture of how the people actually feel about the program.

Students could find a variety of examples, from newspapers or the Internet, such as quality control or public opinion polls, in which confidence levels, confidence intervals and margins of error are used to report results. They should interpret the meaning of the confidence intervals and levels and its implications in society. These examples could then be used to prompt discussions around interpreting and explaining statistical data leading to forming an opinion on the topic.

General Outcome: Develop statistical reasoning.

Suggested Assessment Strategies

Paper and Pencil

- The results of a survey show that 72% of residents in Mount Pearl own cell phones. The margin of error for the survey was 2.3%. If there are 24 600 people in Mount Pearl, ask students to determine the range of the number of people that own cell phones.
(S2.3)

Performance/Observation

- Ask students to create a poster which includes an example from the print or electronic media that uses confidence intervals or confidence levels to support a position. Students could be asked to respond to the following questions:
  (i) Interpret the confidence interval and confidence levels for someone who has no knowledge of statistics.
  (ii) How would the confidence interval change if the sample size used for the study doubled?
  (iii) Do you agree or disagree with any concluding statements that were made about the data from the study? Explain.
(S2.4, S2.5, S2.6))

- Ask students to find an example in the media which uses confidence intervals or confidence levels to support a position relevant to teenagers today. Write two short news articles on the topic. In one article, make the situation sound as disastrous as you can. In the other article, try to minimize the problem.
(S2.4, S2.5, S2.6)

Resources/Notes

Authorized Resource

Principles of Mathematics 11

5.6: Confidence Intervals

SB: pp.295-304
TR: pp.282-288

Quadratic Functions

Suggested Time: 16 Hours

Unit Overview

Focus and Context

In this unit, students will be introduced to the three forms of the quadratic function: standard form, factored form and vertex form. They will investigate the characteristics of quadratic function using features such as *x* and *y*-intercepts, vertex, axis of symmetry, domain and range.

Students will have an opportunity to make the connection between the equation of a quadratic function to its graph. Each form of a quadratic has its own characteristics, and its own benefits. Students will not be expected to use the method of completing the square to convert the vertex form of the quadratic function to standard form. They will however, convert the vertex form of the function to standard form.

Outcomes Framework

GCO

Develop algebraic and graphical reasoning through the study of relations.

SCO RF1

Demonstrate an understanding of the characteristics of quadratic functions, including:

- vertex
- intercepts
- domain and range
- axis of symmetry.

SCO RF2

Solve problems that involve quadratic equations.

GCO

Develop number sense and logical reasoning.

SCO NL2

Analyze puzzles and games that involve spatial reasoning, using problem-solving strategies.

Process Standards Key

[C] Communication	[PS] Problem Solving
[CN] Connections	[R] Reasoning
[ME] Mental Mathematics and Estimation	[T] Technology
	[V] Visualization

SCO Continuum

Mathematics 1201	Mathematics 2201	Mathematics 3201
Relations and Functions	Relations and Functions	Relations and Functions
RF5 Determine the characteristics of the graphs of linear relations, including the: • intercepts • slope • domain • range. [CN, PS, R V]	RF1 Demonstrate an understanding of the characteristics of quadratic functions, including: • vertex • intercepts • domain and range • axis of symmetry. [CN, PS, T, R] RF2 Solve problems that involve quadratic equations. [C, CN, PS, R, T, V]	RF7 Represent data, using polynomial functions (of degree ≤ 3), to solve problems. [C, CN, PS, T, V]
	Number and Logic	Logical Reasoning
	NL2 Analyze puzzles and games that involve spatial reasoning, using problem-solving strategies. [CN, PS, R, V]	LR1 Analyze puzzles and games that involve numerical and logical reasoning, using problem-solving strategies. [CN, ME, PS, R]

Relations and Functions

Outcomes

Students will be expected to

RF1 Demonstrate an understanding of the characteristics of quadratic functions, including:

- vertex
- intercepts
- domain and range
- axis of symmetry.

[**CN, PS, T, R**]

Achievement Indicator:

RF1.1 *Determine the characteristics of a quadratic function ($y = ax^2 + bx + c, a \neq 0$) through manipulation of the parameters a, b and c.*

Elaborations—Strategies for Learning and Teaching

In Grade 9, students were introduced to the concept of degree through the study of polynomials (9PR5). In Mathematics 1201, students factored differences of squares, perfect square trinomials and polynomials of the form $x^2 + bx + c$ and $ax^2 + bx + c$ (AN5). They were also introduced to functional notation through work with linear functions (RF8). In this unit, students will be introduced to quadratic functions. They will examine quadratic functions expressed in the following three forms and will sketch graphs using characteristics such as x and y-intercepts, vertex (as a maximum or minimum point), axis of symmetry and domain and range.

Standard Form: $f(x) = ax^2 + bx + c$

Factored Form: $f(x) = a(x - r)(x - s)$

Vertex Form: $f(x) = a(x - h)^2 + k$

Students will explore each form of the quadratic function independently.

Before students are exposed to the standard form of a quadratic, they need to become familiar with the shape of a quadratic function and how to identify a quadratic function. The terms *quadratic* and *parabola* are new to students. This will be their first exposure to functions that are non-linear.

Students should have an opportunity to investigate what makes a quadratic function. Have them turn to a partner to multiply two linear equations or square a binomial of the form $ax + b$. Consider the examples $y = (x + 1)(x - 4)$ and $y = (3x - 2)^2$. Ask students what they notice in terms of the degree of the polynomial.

Projectile motion can be used to explain the path of a baseball or a skier in flight. To help students visualize the motion of a projectile, toss a ball to a student. Ask students to describe the path of the ball to a partner and have them sketch the path of the height of the ball over time (the independent axis represents time and the dependent axis represents the height of the ball). Encourage them to share their graphs with other students. Ask students to think of other examples that might fit the diagrams of parabolas that open upward or downward.

Characteristics of the resulting parabola, such as vertex and axis of symmetry, should be discussed. It is important for them to recognize that each point on a parabola, except the vertex, has a corresponding point on its mirror image, resulting in the parabolic shape. Discussions around everyday applications like projectile motion give students an appreciation of the usefulenss of quadratics.

General Outcome: Develop algebraic and graphical reasoning through the study of relations.

Suggested Assessment Strategies

Paper and Pencil

- Ask students to make a conjecture about how they could determine whether a quadratic function has either a maximum or minimum value without graphing.

(RF1.1)

- Provide the following table with a set of statements. Ask students to describe the reasoning they used to decide whether each statement is true or false.

Polynomial Function	Classification	True or False	Explain/Justify
$y = 5(x + 3)$	Linear		
$y = 5(x^2 + 3)$	Quadratic		
$y = 5^2(x + 3)$	Quadratic		
$y = 5x(x + 3)$	Linear		
$y = (5x + 1)(x + 3)$	Quadratic		
$y = 5(x + 3)^2 + 2$	Quadratic		

(RF1.1)

Resources/Notes

Authorized Resource

Principles of Mathematics 11

6.1: Exploring Quadratic Relations

Student Book(SB): pp.322-324
Teacher Resource(TR): pp.315-319

Relations and Functions

Outcomes

Students will be expected to

RF1 Continued ...

Achievement Indicator:

RF1.1 *Continued*

Elaborations—Strategies for Learning and Teaching

Students will first explore quadratic functions in standard form. Students should be introduced to the quadratic term, linear term and constant of a quadratic equation. They should distinguish between the quadratic term and the coefficient of the quadratic term. Students often mistake the quadratic term, for example, as the value of a instead of ax^2.

The characteristics of the quadratic function, $f(x) = ax^2 + bx + c$ should be developed through an investigation where the parameters a, b, and c are manipulated individually. Technology, such as graphing calculators, FX Draw or other suitable graphing software, should be used as students' graphing abilities for quadratics would be limited to the use of a table of values at this point. Consider an activity such as the following to examine the effects of manipulating the values of a, b, and c.

Students will first investigate the effect of changing the value of a by comparing quadratic functions $y = x^2$ and $y = ax^2$. As they compare the graphs of $y = 2x^2$ and $y = -2x^2$ to the graph of $y = x^2$, use prompts such as the following to promote student discussion:

- What happens to the direction of the opening of the quadratic if $a < 0$ and $a > 0$?
- If the quadratic opens upward, is the vertex a maximum or minimum point? Explain your reasoning. What if the quadratic opens downward?
- Is the shape of the parabola effected by the parameter a ? Are some graphs wider or narrower compared to the graph of $y = x^2$?
- What happens to the x-intercepts as the value of a is manipulated?
- What is the impact on the graph if $a = 0$?

Similarly, students can compare the graphs of $y = \frac{1}{2}x^2$ and $y = -\frac{1}{2}x^2$ to the graph of $y = x^2$.

General Outcome: Develop algebraic and graphical reasoning through the study of relations.

Suggested Assessment Strategies

Journal

- Jack was asked to describe how the graph of $y = \frac{1}{2}x^2$ compares to the graph of $y = x^2$. He said that the graph of $y = \frac{1}{2}x^2$ will be narrower than the graph of $y = x^2$. Ask students if Jack is correct? Explain your reasoning.

(RF1.1)

- Ask student to explain how changing the parameter a affects the graph of the function $f(x) = ax^2 + bx + c$.

(RF1.1)

Resources/Notes

Authorized Resource

Principles of Mathematics 11

6.1: Exploring Quadratic Relations

SB: pp.322-324
TR: pp.315-319

Web Link

The following website explores the characteristics of a quadratic function through manipulating parameters *a*, *b*, and *c*.
http://www.mathsisfun.com/algebra/quadratic-equation-graph.html

iPhone/iPod apps are available to help students see graphing characteristics.

- Free Graphing Calculator by William Jockusch
- Quadratic Master by Glimsoft (includes tutorials, quadratic equation solver, and other features which can be used throughout this unit)

Relations and Functions

Outcomes

Students will be expected to

RF1 Continued ...

Achievement Indicators:

RF1.1 *Continued*

RF1.2 *Determine, with or without technology, the coordinates of the vertex of the graph of a quadratic function.*

Elaborations—Strategies for Learning and Teaching

Students should then proceed to examine the effects of manipulating the value of b and c in a similar manner. They should be given time to analyze the graphs of various quadratic functions, such as $y = x^2 + 2x$, $y = x^2 + 5x$, $y = x^2 + 1$, and $y = x^2 - 3$. Encourage them to look for connections between the changes in the graph and the function relative to $y = x^2$. Ask questions such as the following:

- What is the effect of parameter b in $y = x^2 + bx$ on the graph of $y = x^2$? Is the parabola's line of symmetry changing?
- What is the effect of parameter c in $y = x^2 + c$ on the graph of $y = x^2$? How can you identify the y-intercept from the equation in standard form? Is the line of symmetry affected by the parameter c?

This activity allows students to recognize that the coefficients of a quadratic function determines the shape of the graph and its orientation.

Students can determine the coordinates of the vertex of a quadratic function with and without technology. The graph of a quadratic function can be created using technology, such as FX Draw or a graphing calculator. Students can more easily identify the characteristics of the quadratic function, such as the coordinates of the vertex, direction of opening, and the x and y-intercepts, when they have a visual representation.

Another option is to create a table of values. In Grade 9, students used a table of values to graph linear relations. They will now extend this strategy to quadratic equations to determine the vertex and its connection to the axis of symmetry. When graphing the quadratic function $y = x^2 + 4x + 3$, for example, students can create the following table of values.

x	-4	-3	-2	-1	0	1	2	3
y	3	0	-1	0	3	8	15	24

Ask students if they can recognize from this table, the point at which the parabola changes direction. The goal is for students to be able to identify the symmetry since the points on the parabola that share the same y-coordinate are equidistant from the vertex.

Whether students use a graph or a table of values, teachers should promote discussion by asking questions such as the following:

- What connection is there between the axis of symmetry and the coordinates of the vertex?
- What is the equation of the axis of symmetry?

General Outcome: Develop algebraic and graphical reasoning through the study of relations.

Suggested Assessment Strategies

Performance

- Ask students to work in groups for this activity. Each group should be given a pair of dice (or they can create their own). On the first die, there will be 2 *a* values, 2 *b* values, and 2 *c* values. On the second die, there will be various rational numbers. Students should roll the dice and state the effect on $y = x^2$ of changing the given parameter to the given number.

(RF1.1)

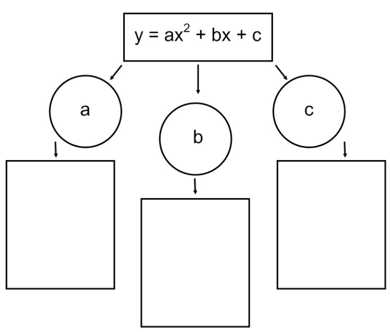
Paper and Pencil

- Ask students to compare the graphs of the following equations to the graph of $y = x^2$:

  (i) $y = 3x^2 - 1$
  (ii) $y = 2x^2 + x$
  (iii) $y = -\frac{1}{2}x^2 - 4x + 3$

(RF1.1)

- Ask students to complete the webbing to describe the effects of the parameters *a*, *b* and *c* on the quadratic function $y = ax^2 + bx + c$.



(RF1.1)

- Ask students to graph each of the following using a table of values or graphing technology and record changes in the parabola as the parameter *c* is manipulated.

  (i) $y = x^2$
  (ii) $y = x^2 + 2$
  (iii) $y = x^2 + 5$
  (iv) $y = x^2 - 3$
  (v) $y = x^2 - 4$

(RF1.1)

Resources/Notes

Authorized Resource

Principles of Mathematics 11

6.1: Exploring Quadratic Relations

SB: pp.322-324
TR: pp.315-319

6.2: Properties of Graphs of Quadratic Functions

SB: pp.325-336
TR: pp.320-324

Relations and Functions

Outcomes

Students will be expected to

RF1 Continued ...

Achievement Indicators:

RF1.2 *Continued*

RF1.3 *Determine the coordinates of the vertex of the graph of a quadratic function, given the equation of the function and the axis of symmetry, and determine if the y-coordinate of the vertex is a maximum or a minimum value.*

Elaborations—Strategies for Learning and Teaching

Students may use the formula $x = -\frac{b}{2a}$ to find the *x*-coordinate of the vertex for quadratic functions of the form $y = ax^2 + bx + c$. This value can then be used to determine the equation of the axis of symmetry. The formula should be developed through the student based activity outlined below.

Provide students with a table containing quadratic functions in standard form and the vertex of each function. Students should identify the values of *a*, *b* and *c* along with the value of $-\frac{b}{2a}$.

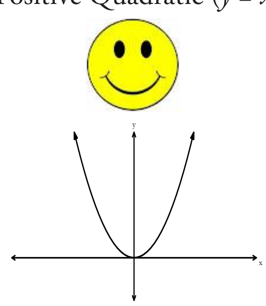
Function	Vertex	Equation of Axis of Symmetry	a	b	c	$-\frac{b}{2a}$
$y = x^2 - 4x + 7$	(2, 3)					
$y = -2x^2 - 16x - 34$	(-4, -2)					
$y = 3x^2 - 6x + 10$	(1, 7)					

Students should discover that the value of $-\frac{b}{2a}$ is the *x*-coordinate of the vertex, and the connection should be made to the equation of the axis of symmetry. The *y*-coordinate of the vertex can be found by substitution of the *x*-coordinate into the quadratic function. Encourage students to work with and compare the various methods and decide which method they prefer and explain why.

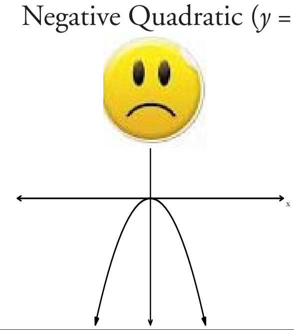
If the axis of symmetry is provided, in addition to the standard form of the quadratic function, ask students how they could determine the *y*-coordinate of the vertex. Teachers could start the discussion by reminding students of their work with linear relations in Mathematics 1201. When given the slope and the coordinates of a point on the line, students determined the *y*-intercept through the use of substitution. This algebraic method can also work with quadratics by substituting the known *x*-value into the function to find the corresponding *y*-value.

The idea of the vertex being a maximum or minimum point is determined by the value of *a*. One way to help students remember how to determine direction of opening is shown here.

Positive Quadratic ($y = x^2$)



Negative Quadratic ($y = -x^2$)



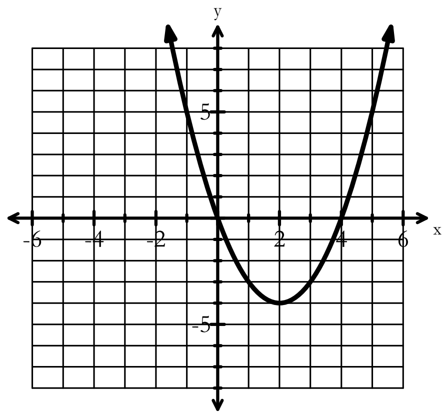
General Outcome: Develop algebraic and graphical reasoning through the study of relations.

Suggested Assessment Strategies

Paper and Pencil

- Ask students to determine the vertex of each of the following:

  (i)



(ii)

x	-1	0	1	2	3
y	10	1	-2	1	10

(iii) $y = 2x^2 - 8x + 7$

(RF1.2)

Resources/Notes

Authorized Resource

Principles of Mathematics 11

6.2: Properties of Graphs of Quadratic Functions

SB: pp.325-336
TR: pp.320-324

Relations and Functions

Outcomes

Students will be expected to

RF1 Continued ...

Achievement Indicator:

RF1.3 *Continued*

RF1.4 *Determine the equation of the axis of symmetry of the graph of a quadratic function using the x-intercepts of the graph.*

Elaborations—Strategies for Learning and Teaching

This can provide useful information, particularly when sketching graphs of quadratic functions. If students plot points resulting in a quadratic opening upward, but the coefficient of the quadratic term of the function is negative, this is a good indicator that something is wrong with their graph.

It is important for students to distinguish the difference between a maximum or minimum point and a maximum or minimum value.

Up to this point, students have determined the equation of the axis of symmetry using the x-coordinate of the vertex. The axis of symmetry can also be linked to the x-intercepts of the graph of a quadratic function. Provide students with several graphs of quadratic functions and ask them how the location of the x-intercepts and the axis of symmetry are connected. The focus is for students to recognize that, due to symmetry, the axis of symmetry is exactly half-way between the two x-intercepts.

Students should also be given an opportunity to analyze several tables of values and their corresponding graphs. Use the following questions to help students make a connection between the axis of symmetry and x-intercepts:

- How are the x-intercepts determined using a table of values?
- Can you identify the vertex from the table? Explain.
- How could you determine the x-cordinate of the vertex just from the x-intercepts?
- What happens if the x-intercepts are not evident in a table of values? What strategy can be used to find the axis of symmetry?

It is important for students to recognize that given any two points with the same y-coordinates in a table of values, the equation of the axis of symmetry can be determined by averaging the x-coordinates of the points.

Students sometimes incorrectly state the equation of the axis of symmetry as a numerical value rather than an equation. It is important for them to recognize that, when they are graphing quadratic functions, the line of reflection is a vertical line. For the quadratic function, $y = x^2 - 6x + 13$, students may mistakenly identify the equation of the axis of symmetry as 3 rather than $x = 3$.

General Outcome: Develop algebraic and graphical reasoning through the study of relations.

Suggested Assessment Strategies

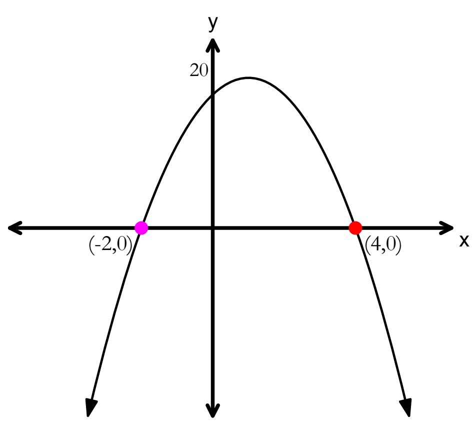
Interview

- Given the graph of $y = -2x^2 + 8x - 3$, ask the following interview questions:
  - (i) What is the value of the y-intercept? x-intercept?
  - (ii) What is the direction of opening?
  - (iii) What is the vertex?
  - (iv) What is the equation of the axis of symmetry?
  - (v) What is the connection between the axis of symmetry and the vertex?

(RF1.2, RF1.3, RF1.4)

Paper and Pencil

- Ask students to find the equation of the axis of symmetry for the parabola shown in the graph below.



(RF1.4)

Resources/Notes

Authorized Resource

Principles of Mathematics 11

6.2: Properties of Graphs of Quadratic Functions

SB: pp.325-336
TR: pp.320-324

Relations and Functions

Outcomes

Students will be expected to

RF1 Continued ...

Achievement Indicator:

RF1.4 *Continued*

Elaborations—Strategies for Learning and Teaching

Students can also use some of the following techniques to help them determine the equation of the axis of symmetry of non-factorable quadratic functions until the quadratic formula is developed in the next unit. The zero product property will have to be introduced, but a more detailed discussion will occur in the next unit.

One method involves using a transformational approach. For example, $f(x) = 2x^2 - 4x + 7$ is not factorable but a vertical translation of -7 would produce the function $f(x) = 2x^2 - 4x$, which is factorable. Since a vertical translation of a quadratic function has no effect on the equation of the axis of symmetry, both functions will have the same equation of the axis of symmetry. The x-intercepts of this function can be used to determine the equation of the axis of symmetry of both functions.

$f(x) = 2x^2 - 4x$

$2x(x - 2) = 0$

$2x = 0$ and $x - 2 = 0$

$\therefore x = 0 \quad x = 2$

The equation of the axis of symmetry is $x = 1$.This is a good opportunity to connect line symmetry to the manipulation of the parameter c in $f(x) = ax^2 + bx + c$, which was introduced at the beginning of this unit.

Students have worked with a table of values where the points share the same y-coordinate. Since these points are equidistant from the vertex, the equation of the axis of symmetry is located exactly halfway between them. The method of partial factoring will produce the same result. Consider the following example:

$f(x) = 2x^2 - 4x + 7$	not factorable
$f(x) = 2x(x - 2) + 7$	partial factoring strategy
$2x = 0$ and $x - 2 = 0$	Zero Product Property
$x = 0 \quad x = 2$	
$\therefore f(0) = 7$ and $f(2) = 7$	(0, 7) and (2, 7) are points on the graph with the same y-coordinate

The equation of the axis of symmetry is located half way between 0 and 2. The equation of the axis of symmetry, therefore, is $x = 1$. When using this method, remind students that they are not determining x-intercepts here as they would if they were factoring the quadratic function. A graphical representation may help students visualize the information this method provides.

General Outcome: Develop algebraic and graphical reasoning through the study of relations.

Suggested Assessment Strategies

Observation

- This activity will involve a group of two students. Give one student the graph of a quadratic function. Ask him/her to turn to their partner and describe, using the characteristics of the quadratic function, the graph they see. The other student will draw the graph based on the description from the student. Both students will then check to see if their graphs match.

(RF1.2, RF1.3, RF1.4)

Resources/Notes

Authorized Resource

Principles of Mathematics 11

6.2: Properties of Graphs of Quadratic Functions

SB: pp.325-336
TR: pp.320-324

6.3: Factored Form of a Quadratic Function

SB: pp.337-353
TR: pp.325-332

Web Link

www.k12pl.nl.ca/seniorhigh/introduction/math2201/classroomclips.html

The **Describe and Draw** clip related to *Characteristics of Quadratic Functions* demonstrates students graphing a quadratic function by listening to the information presented by their partner.

Relations and Functions

Outcomes

Students will be expected to

RF1 Continued ...

Achievement Indicators:

RF1.5 *Sketch the graph of a quadratic function.*

RF1.6 *Determine the domain and range of a quadratic function.*

Elaborations—Strategies for Learning and Teaching

Encourage students to think about what the graph of the quadratic function should look like before actually sketching the graph. They can discuss features such as the direction of the opening, whether the vertex is a maximum or minimum point, what the value of the y-intercept is, and how many x-intercepts there should be. Consider, for example, the equation $y = -x^2 + 5x + 4$. Since the parabola opens downward (a value is negative), the vertex is a maximum point. The y-intercept (c value) is 4, which is above the x-axis, resulting in two x-intercepts.

Ask students what are the important characteristics to consider when sketching the graph of a quadratic function. Students may intially use a table of values to draw their sketch. Encourage them to use the other methods addressed throughout this unit. For example, students may use the fomula $-\frac{b}{2a}$ to find the x-coordinate of the vertex and use substitution to find the corresponding y-value. To draw an accurate graph, students should plot at least two other points on the graph.

Students will be required to determine the domain and range of the function. The concept of domain and range was introduced in Mathematics 1201 using set notation or interval notation (RF5). Students should recognize that all non-contextual quadratic functions have a domain of $\{x| \ x \in \Re\}$, whereas the range depends on the vertex and the direction of the opening. Teachers should promote discussion when students are graphing quadratic functions by asking the following questions:

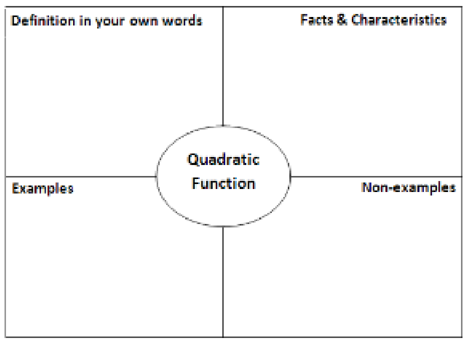
- Why is the domain the set of all real numbers when only some points are plotted from the table of values?
- How is the range related to the direction of the opening?

General Outcome: Develop algebraic and graphical reasoning through the study of relations.

Suggested Assessment Strategies

Paper and Pencil

- Ask students to complete the following Frayer model.



(RF1.2, RF1.3, RF1.4, RF1.5, RF1.6)

Resources/Notes

Authorized Resource

Principles of Mathematics 11

6.2: Properties of Graphs of Quadratic Functions

SB: pp.325-336
TR: pp.320-324

6.3: Factored Form of a Quadratic Function

SB: pp.337-353
TR: pp.325-332

Relations and Functions

Outcomes

Students will be expected to

RF2 Solve problems that involve quadratic equations.
[C, CN, PS, R, T, V]

Achievement Indicator:

RF2.1 *Determine, with or without technology, the intercepts of the graph of a quadratic function.*

Elaborations—Strategies for Learning and Teaching

Students will work with quadratic functions in factored form. They will make the connection between the factored form of a quadratic function and the *x*-intercepts of its graph. Consider the following activity:

Using technology, ask students to graph the following functions and determine the *x*-intercepts:

(i) $y = x^2 - 6x + 9$

(ii) $y = 2x^2 - 4x - 6$

(iii) $y = (x - 2)(x + 3)$

(iv) $y = (x + 1)(x - 4)$

Lead a discussion focussing on the following questions:

- Can you determine the *x*-intercepts by looking at a quadratic function? Explain.
- Which form of the quadratic function did you find the easiest to use when determining the *x*-intercepts?
- What is the connection between the factors and the *x*-intercepts.
- What is the value of the *y*-coordinate at the point where the graph crosses the *x*-axis?
- What would happen if the factors of the quadratic function were identical?
- How can you find the *x*-intercepts of a quadratic function without the graph or a table of values?

Students will be required to use factoring methods developed in Mathematics 1201 to determine the zeros. It is important for students to distinguish between the terms zeros and *x*-intercepts, and to use the correct term in a given situation. The *zero product property* states that if the product of two real numbers is zero, then one or both of the numbers must be zero. This will be new for students. It is important for them to make the connection that the values of *x* at the zeros of the function are also the *x*-intercepts of the graph.

A common student error occurs when students factor a quadratic and mistakenly ignore the signs when determining the *x*-intercepts.For example, when students factor $y = x^2 + 2x - 8$ as $y = (x + 4)(x - 2)$, they often state the *x*-intercepts are 4 and -2. Reinforce to students that they are solving equations $x + 4 = 0$ and $x - 2 = 0$ to obtain the *x*-intercepts -4 and 2.

General Outcome: Develop algebraic and graphical reasoning through the study of relations.

Suggested Assessment Strategies

Journal

- Ask students how they can determine the coordinates of the vertex given the factored form of a quadratic function.

(RF2.1

Performance

- *Quiz-Quiz-Trade* is a group activity for the entire class. Questions and answers are written on index cards (see sample cards below).To begin, distribute one card to each student. Ask students to choose partners. Student #1 quizzes Student #2. If Student #2 answers correctly, Student #1 gives positive feedback. If Student #2 answers incorrectly, Student #1 provides direction on how to get the correct answer. Then Student #2 quizzes Student #1. After they both quiz each other with their questions, they switch/trade their questions and go on to pair up with another student. This process should be repeated at least 5 times.

Question: What is the range of the quadratic function $y=2(x-4)^2+1$? Answer: $\{y \mid y \geq 1, y \in R\}$	Question: What is the vertex of the quadratic function $y=-x^2-4x+3$? Answer: $(-2,7)$

(RF1.2, RF1.3,RF1.4, RF1.6, RF2.1)

Resources/Notes

Authorized Resource

Principles of Mathematics 11

6.3: Factored Form of a Quadratic Function

SB: pp.337-353
TR: pp. 325-332

Relations and Functions

Outcomes

Students will be expected to

RF2 Continued ...

Achievement Indicators:

RF2.1 *Continued*

RF2.2 *Express a quadratic equation in factored form, given the zeros of the corresponding quadratic function or the x-intercepts of the graph of the function.*

Elaborations—Strategies for Learning and Teaching

When students were introduced to common factors and trinomial factoring, they modelled the factoring concretely and pictorially and recorded the process symbolically. Some students may continue to benefit from concrete representations. Therefore, manipulatives, such as algebra tiles, should be available for use.

Whether the equation is written in standard form or factored form, once students have determined the x-intercepts, they can continue to find the vertex and the y-intercept. Using this information, students can graph the quadratic function.

Students have been exposed to the graph of a quadratic and the points where a parabola crosses the x-axis. They will now use the x-intercepts and an additional point to determine the equation of a parabola. This is a process where students work backwards. Seeing a question starting with the quadratic function and ending with the zeros might help students see how to start with the zeros and end with the quadratic function.

Students should experiment with parabolas where the x-intercepts are provided. Promote student discussion around the following activity:

- Sketch a parabola that passes through (-4,0) and (3,0).
- Make three more parabolas that are different from the first one, but still have x-intercepts -4 and 3.
- How many parabolas do you think are possible? Explain.

The goal is for students to recognize that a family of parabolas are possible when the x-intercepts are given. When provided with an extra point, however, students can narrow down the exact formula for the quadratic equation. In order to determine the multiplier a in the factored form $y = a(x - r)(x - s)$, students will choose any point on the parabola and use substitution. They should compare their answers with the class to confirm that the points they chose resulted in the same quadratic function, and then have a discussion about whether some points are easier to work with than others. Using the distributive property, students may write the equation from factored form to standard form.

General Outcome: Develop algebraic and graphical reasoning through the study of relations.

Suggested Assessment Strategies

Paper and Pencil

- Ask students to determine the quadratic function with factors (x + 3) and (x - 5) and a y-intercept of -5.

(RF2.2)

- Ask students to respond to the following questions about quadratic functions:

  (i) Are two x-intercepts enough information to sketch a unique graph? Explain.

  (ii) Is the vertex enough information to sketch a unique graph? Explain.

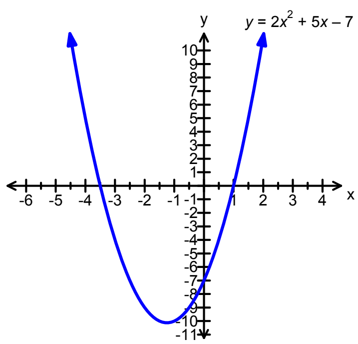
  (iii) What are the minimum requirements to sketch a unique quadratic graph? Explain.

(RF2.2)

- Ask students to answer the following questions:

  (i) What are the zeros of $f(x) = 2x^2 + 5x - 7$?

  (ii) What are the x-intercepts of the graph?



  (iii) What do you notice about the answers to the above questions?

(RF2.1)

Resources/Notes

Authorized Resource

Principles of Mathematics 11

6.3: Factored Form of a Quadratic Function

SB: pp.337-353
TR: pp. 325-332

Relations and Functions

Outcomes

Students will be expected to

RF1 Continued ...

Achievement Indicators:

RF1.2, RF1.3

RF1.5, RF1.6 *Continued*

Elaborations—Strategies for Learning and Teaching

Each form of a quadratic has its own characteristics, and its own benefits. If the quadratic is written is standard form, for example, students can determine the y-intercept and the direction of the opening of the parabola directly from the equation. If the equation is written in factored form, they can determine the x-intercepts of the graph and the direction of the opening of the parabola. Students will now look at the adavantage of writing a quadratic in vertex form.

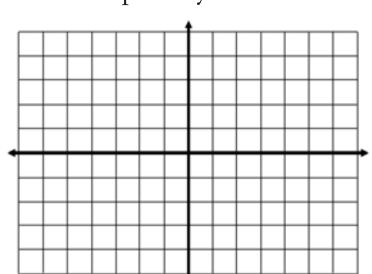
Students will graph a quadratic function in vertex form $y = a(x - h)^2 + k$, and relate the characteristics of the graph to its function. Students were exposed to parameter a earlier in this unit. They will now explore the components h and k. Using graphing technology, students can compare graphs such as:

$y = x^2$ and $y = (x + 2)^2$
$y = x^2$ and $y = x^2 + 3$
$y = x^2$ and $y = (x - 1)^2 - 5$

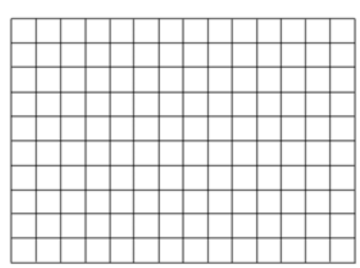
The focus is to observe the vertical translation, k, and the horizontal translation, h, to make a prediction about the vertex (h, k) of each equation. When given the equations $y = x^2$ and $y = x^2 + 4$, for example, students make the connection that the original y- values are increasing by 4 but the x-values remain unchanged. The positive, therefore, means upward. Students sometimes have more conceptual problems with horizontal transformations. Consider the equations $y = x^2$ and $y = (x + 4)^2$. How do we explain to students that the horizontal translation is the opposite operation inside the parenthesis?

In Mathematics 1201, students represented functions using function notation. The concept was introduced as a rule that takes an input value and gives a unique corresponding output value. The following activity relating input and output to vertical and horizontal translations of functions may be worthwhile.

- Provide students with an axis on plain paper and a grid on the transparency as shown.



Input (Paper)



Output (Transparency)

General Outcome: Develop algebraic and graphical reasoning through the study of relations.

Suggested Assessment Strategies

Performance

- *The Quadratic Round Table*: Students will graph quadratic equations using a structure called "Roundtable".
    (i) Organize students into teams of 4 with desks facing each other.
    (ii) One sheet of paper is used per function, per team.
    (iii) Provide a quadratic function that is to be graphed by the team.
    (iv) The goal is to have each team graph the assigned quadratic, one step at a time. Each student takes a turn stating a characteristic (vertex, equation of the axis of symmetry, *x*-intercepts, *y*-intercepts and domain and range) and then passes the paper to the next member of the team.
    (v) Incorrect responses should be noted and corrected by other team members. This constitutes one turn for that member.
    (vi) When the team is satisfied that the quadratic has been correctly graphed they will sketch their final answer on the board. Answers can then be compared as a class.

(RF1.1, RF1.2, RF1.3, RF1.4, RF1.5, RF1.6, RF2.1)

- *Find your partner*: Provide half the students with cards which show graphs labelled with the vertex or *x*-intercepts. The other half of the class will be given cards which have the corresponding quadratic functions in a variety of forms. Students move around the classroom trying to locate their matching card.

(RF1.2, RF1.3, RF1.4, RF1.5)

Journal

- Based on the following information, ask students to determine which form of the quadratic equation they would prefer to write and why.
    (i) the vertex of the parabola is (5, -2) and passes through the point (-1, -4)
    (ii) the factors are (x - 1) and (x + 3) and passes through the point (-2, 3)

(RF1.2, RF1.3, RF1.4, RF1.5, RF2.2)

Resources/Notes

Authorized Resource

Principles of Mathematics 11

6.4: Vertex Form of a Quadratic Function

SB: pp.354-367
TR: pp.333-338

Relations and Functions

Outcomes

Students will be expected to

RF1 Continued ...

Achievement Indicators:

RF1.2, RF1.3

RF1.5, RF1.6 *Continued*

Elaborations—Strategies for Learning and Teaching

- Ask students to lay the transparency over the axis and sketch the graph of $y = x^2$ on the transparency.
- Students first need to identify the transformation as output (k) or input (h). The graph on the transparency represents the output values of the function, whereas the x-axis on the plain paper represents the input values. Using this visual aid, students should recognize that an output transformation will move the graph, and an input transformation will move the grid.
- Provide students with several translations of $y = x^2$ such as:

  (i) $f(x) + 3 = x^2 + 3$
  (ii) $f(x - 4)=(x - 4)^2$
  (iii) $f(x + 1) - 2=(x + 1)^2 - 2$
- When working with $f(x) + 3 = x^2 + 3$, students should identify it as output transformation. Ask students which of the materials will be moved and what do they notice? If the output is affected, motion is on the transparency, not on the plain paper. Using this visual aid, students should recognize that the graph shifts upward 3 units. Similarly, ask students to explore $f(x) - 1$.
- When working with $f(x - 4) = (x - 4)^2$, students should identify it as input transformation. The input is being decreased by 4. This corresponds to every x-value decreasing by 4 before being inputed into the function. Ask students if the transparency or the plain paper moves. Students will move the plain paper 4 units to the left underneath the transparency containing the graph of $y = x^2$. Observe students while they do this activity and ask them how the graph shifts. Students should recognize that moving the axis results in the opposite effect on the graph. That is $(x - 4)^2$ translates the graph 4 units right. Similarly, ask students to explore $f(x+1)$?

Once students have determined the vertex directly from the function, they can sketch the graph using the vertex, the axis of symmetry, the y-intercept and the direction of opening of the parabola, without using a table of values. Students can convert the vertex form of the function to standard form. They will not be expected to use the method of completing the square to convert the standard form of the quadratic function to vertex form.

General Outcome: Develop algebraic and graphical reasoning through the study of relations.

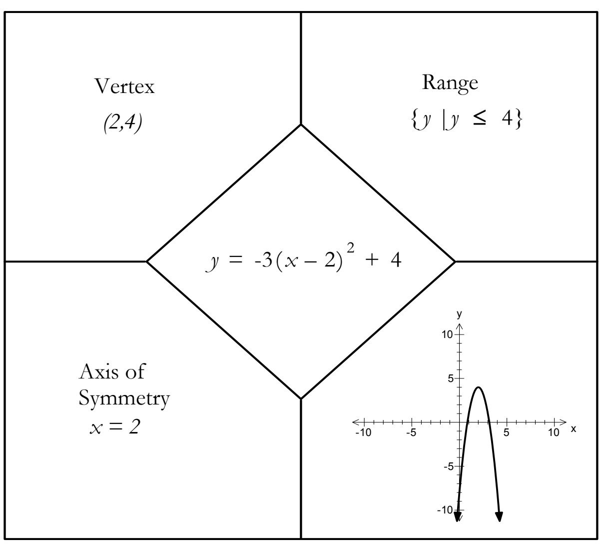
Suggested Assessment Strategies

Observation

- This activity will involve a group of three students. Provide students with a completed Frayer model cut into five puzzle pieces. To make this more challenging some of the puzzle pieces remain blank. The titles would be Graph, Table of Values, Equation, Characteristics, and Domain and Range. Each student is randomly given one piece of the quadratic function and is to move about the classroom locating the corresponding pieces to put the puzzle back together and complete the Frayer model. Once the group is formed, they will have three pieces of the puzzle and as a group will have to complete the remaining two pieces. Teachers can first use this model with standard form and then apply it to the vertex and factored form of a quadratic.

(RF1.2, RF1.3, RF1.4, RF1.5, RF1.6, RF2.1)

- Students can work in pairs to complete the following quadratic puzzle investigating the characteristics and graphs of various quadratic functions of the form $y = a(x - p)^2 + q$. They should work with 20 puzzle pieces (4 complete puzzles consisting of a function and four related characteristics) to correctly match the characteristics with each function. A sample is shown below.



(RF1.2, RF1.3, RF1.4, RF1.5, RF1.6, RF2.1)

Resources/Notes

Authorized Resource

Principles of Mathematics 11

6.4: Vertex Form of a Quadratic Function

SB: pp.354-367
TR: pp.333-338

Web Link

www.k12pl.nl.ca/seniorhigh/introduction/math2201/classroomclips.html

The **Frayer Model** clip related to *Characteristics of Quadratic Functions* demonstrates students finding the different representations of a quadratic function and completing a puzzle.

Relations and Functions

Outcomes

Students will be expected to

RF1 Continued ...

Achievement Indicators:

RF1.7 *Determine the quadratic function given the characteristics of its graph.*

RF1.8 *Solve a contextual problem that involves the characteristics of a quadratic function.*

RF1.6 *Continued*

Elaborations—Strategies for Learning and Teaching

Students will determine the quadratic function given the vertex and an additional point on the parabola. This would be a good opportunity to review the characteristics of each quadratic form in addition to the limitations of each. Students should be provided with examples where they have to select which form of the quadratic function best suits the information provided. For example, a key piece of information of the vertex form is knowing the vertex, and the benefit of the factored form is knowing the x-intercepts. In both cases, students can then substitute in another point that is on the parabola to determine the value of a. Even though students initially start with the quadratic in vertex or factored form, they can always use their distributive property to rewrite the equation in standard form.

Students should be provided with examples where characteristics of quadratic functions are applied to conceptual problems. Consider the following example:

- A ball is thrown from an initial height of 1 m and follows a parabolic path. After 2 seconds, the ball reaches a maximum height of 21 m. Algebraically determine the quadratic function that models the path followed by the ball, and use it to determine the approximate height of the ball at 3 seconds.

Encourage students to discuss the following:

- How is the shape of the graph connected to the situation?
- What do the coordinates of the vertex represent?
- What do the x and y-intercepts represent?
- Why isn't the domain all real numbers in this situation?

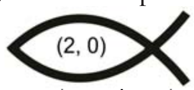
Students should be provided with a variety of contextual problems which place restrictions on the domain and range. To help them distinguish between situations where restrictions on the domain are necessary, students could be exposed to questions such as the following: State the domain and range for the function $f(x) = -0.15x^2 + 6x$. Now suppose the function $h(t) = -0.15t^2 + 6t$ represents the height of a ball, in metres, above the ground as a function of time, in seconds. What impact does this context have on domain and range?

General Outcome: Develop algebraic and graphical reasoning through the study of relations.

Suggested Assessment Strategies

Performance

- *Fishing for Quadratic Functions*: Fill two bags with intercepts (in coordinate form) written on pieces of paper (shaped like fish).



One bag would contain only x-intercepts and the other bag would contain only y-intercepts. Ask students to fish out two x-intercepts and one y-intercept, and then determine the quadratic function that passes through the three points. Note: It will be necessary to have twice as many x-intercepts as y-intercepts.

(RF1.7)

Paper and Pencil

- Ask students to answer the following:

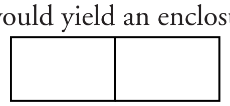
(i) A quarterback throws the ball from an initial height of 6 feet. It is caught by the receiver 50 feet away, at a height of 6 feet. The ball reaches a maximum height of 20 feet during its flight. Determine the quadratic function which models this situation and state the domain and range.

(RF1.6, RF1.7,RF1.8)

(ii) The path of a model rocket can be described by the quadratic function $y = - x^2 - 12x$, where y represents the height of the rocket, in metres, at time x seconds after takeoff. Identify the maximum height reached by the rocket and determine the time at which the rocket reached its maximum height.

(RF1.8)

(iii) You have 600 meters of fencing and a large field. You want to make a rectangular enclosure split into two equal lots. What dimensions would yield an enclosure with the largest area?



(RF1.8)

Observation

- Ask students to play *Parabola Math*. Provide students with a deck of cards containing the graph of the parabola, function of the parabola, vertex of the parabola and the equation of the axis of symmetry. Each characteristic could be on different colour paper. Students should work in groups to match the various components of the parabola.

(RF1.1, RF1.2, RF1.3, RF1.7)

Resources/Notes

Authorized Resource

Principles of Mathematics 11

6.4: Vertex Form of a Quadratic Function

SB: pp.354-367
TR: pp.333-338

6.5: Solving Problems Using Quadratic Function Models

SB: pp.368-382
TR: pp.339-344

Number and Logic

Outcomes

Students will be expected to

NL2 Analyze puzzles and games that involve spatial reasoning, using problem-solving strategies.

[CN, PS, R, V]

Achievement Indicators:

NL2.1 *Determine, explain and verify a strategy to solve a puzzle or to win a game; e.g.,*

- *guess and check*
- *look for a pattern*
- *make a systematic list*
- *draw or model*
- *eliminate possibilities*
- *simplify the original problem*
- *work backward*
- *develop alternative approaches*

Elaborations—Strategies for Learning and Teaching

Invite students to bring in their own games and puzzles that involve spatial reasoning and have not been covered in class. This may involve bringing games or puzzles from home or searching the Internet to find games, puzzles or online applications of interest. As an alternative, students may have a great idea for a puzzle or game that will challenge their classmates. Students will inform the rest of their class about the game/puzzle using questions such as the following:

- What is the puzzle/game and where did you find it?
- Describe the puzzle/game. Why did you select this puzzle/game?
- What is the object of the game and the rules of play?
- What strategy was used to solve the puzzle or win the game?

General Outcome: Develop number sense and logical reasoning.

Suggested Assessment Strategies

Performance

- Using a game or puzzle of their choice, ask students to write their own description of the game/puzzle, including the rules of play, and helpful. They should give the information to another classmate as they play the game/puzzle.

(NL2.1)

Resources/Notes

Authorized Resource

Principles of Mathematics 11

6.5: Solving Problems Using Quadratic Function Models

SB: pp.368-382
TR: pp.339-344

Quadratic Equations

Suggested Time: 16 Hours

Unit Overview

Focus and Context

In this unit, students will solve quadratic equations. They will make the connection that the *x*-intercepts of the graph, or the zeros of the quadratic function correpsond to the solutions, or roots, of the quadratic equation.

Students will solve quadratic equations using various strategies. They may prefer some methods over others depending on the circumstances. Students will be exposed to the concepts of graphing, factoring, quadratic formula or the square root method. They will then solve a contextual problem by modelling a situation with a quadratic equation.

Outcomes Framework

GCO

Develop algebraic and graphical reasoning through the study of relations.

SCO RF2

Solve problems that involve quadratic equations.

Process Standards Key

[C]	Communication	**[PS]**	Problem Solving
[CN]	Connections	**[R]**	Reasoning
[ME]	Mental Mathematics and Estimation	**[T]**	Technology
		[V]	Visualization

SCO Continuum

Mathematics 1201	Mathematics 2201	Mathematics 3201
Algebra and Number	Relations and Functions	Relations and Functions
AN4 Demonstrate an understanding of the multiplication of polynomial expressions (limited to monomials, binomials, and trinomials), concretely, pictorially and symbolically. [CN, R, V] AN5 Demonstrate an understanding of common factors and trinomial factoring, concretely, pictorially and symbolically. [C, CN, R, T]	RF2 Solve problems that involve quadratic equations. [C, CN, PS, R, T, V]	RF7 Represent data, using polynomial functions (of degree ≤ 3), to solve problems. [C, CN, PS, T, V]

Relations and Functions

Outcomes

Students will be expected to

RF2 Solve problems that involve quadratic equations.

[C, CN, PS, R, T, V]

Achievement Indicator:

RF2.1 *Determine, with or without technology, the intercepts of the graph of a quadratic function.*

Elaborations—Strategies for Learning and Teaching

In Mathematics 1201, students factored difference of squares, perfect square trinomials and polynomials of the form $x^2 + bx + c$ and $ax^2 + bx + c$ (AN5).

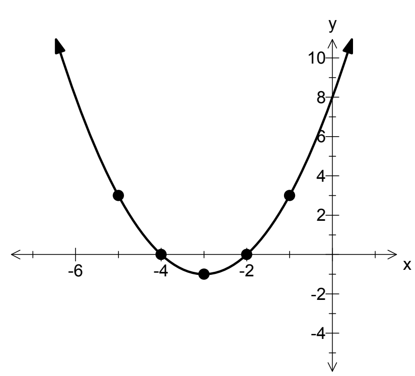
In the previous unit, students were introduced to quadratic functions expressed in standard form, factored form and vertex form. They sketched graphs using characteristics such as x and y-intercepts, vertex, axis of symmetry and domain and range. Students also expressed quadratic functions in factored form relating the x-intercepts of the parabola to the factors of the function.

In this unit, students will solve quadratic equations by graphing, factoring and using the quadratic formula. Choosing the most efficient method is important when solving the quadratic equation.

Discuss with students what it means to solve a quadratic equation graphically. Technology, such as FX Draw, a graphing calculator, or graphing software can be used to graph a quadratic function. Students should be able to identify the intercepts using the visual representation. Provide graphs that model a real-life situation. The focus here is for students to analyze the graph and be able to identify and explain what the x-intercept(s) and y-intercept represent.

Students can use the vertex formula discussed in the previous unit and a table of values to draw a graph. For example, using the formula $x = \frac{-b}{2a}$ students can determine the vertex of $y = x^2 + 6x + 8$ to be (-3, -1). They can then complete a table of values using the vertex as the middle value. Students should be able to use the symmetry of the graph to efficiently complete the table and then sketch the graph.

x	y
-5	3
-4	0
-3	-1
-2	0
-1	3

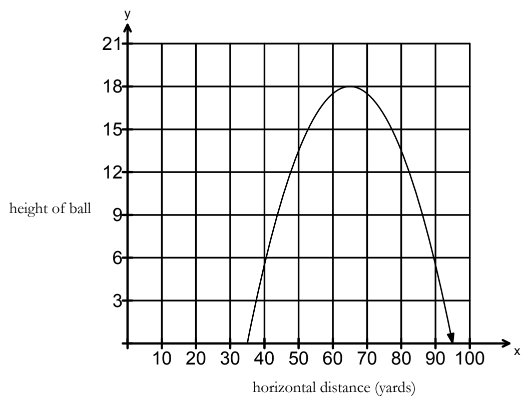


General Outcome: Develop algebraic and graphical reasoning through the study of relations.

Suggested Assessment Strategies

Paper and Pencil

- The path of a football at one particular kick-off can be modelled using the function $h(d) = -0.02d^2 + 2.6d - 66.5$ where h is the height of the ball above the ground (yards) and d is the horizontal distance from the kicking team's goal line (yards).



Ask students to respond to the following:

(i) What are the *x*-intercepts? What do they mean in the context of the problem? Why are there two *x*-intercepts?

(ii) Why is the *y*-intercept not labelled on this graph?

(iii) What horizontal distance does the ball travel before it hits the ground?

(RF2.1)

Resources/Notes

Authorized Resource

Principles of Mathematics 11

7.1: Solving Quadratic Equations by Graphing

SB: pp.396-404
TR: pp.373-381

Relations and Functions

Outcomes

Students will be expected to

RF2 Continued ...

Achievement Indicators:

RF2.1 *Continued*

RF2.3 *Explain the relationships among the roots of an equation, the zeros of the corresponding function, and the x-intercepts of the graph of the function.*

RF2.4 *Explain, using examples, why the graph of a quadratic function may have zero, one or two x-intercepts.*

Elaborations—Strategies for Learning and Teaching

This is an opportunity to discuss the following points:

- Are the the *x*-intercepts exact values? Can they be clearly marked on the graph?
- Can the *x*-intercepts be identified from the table of values?
- How many *x*-intercepts exist? Is this always true?
- What is the *y*-intercept? Can this value be determined from the table of values?
- How can the *x*-intercepts be determined if they are not exact?

It is important for students to realize that not all *x*-intercepts are integers. Whether students draw a graph by hand or use technology, they will approximate the *x*-intercepts using the graph as their visual representation.

The *x*-intercepts of the graph, or the zeros of the quadratic function correspond to the roots of the quadratic equation. Students will be asked to find the roots of the equation $x^2 - 7x + 12 = 0$, find the zeros of $f(x) = x^2 - 7x + 12$, or determine the *x*-intercepts of $y = x^2 - 7x + 12$. In each case they are solving $x^2 - 7x + 12 = 0$ and arriving at the solution $x = 3$ or $x = 4$.

In the previous unit, students studied the graphs of quadratic functions. They made a connection between the coefficient of the quadratic term and the direction of the opening of a quadratic. They also represented the maximum or minimum point of the graph as the vertex. The following activity would be beneficial to have students discover the number of *x*-intercepts that are possible for a quadratic function.

Vertex	Direction of Opening	Graph	Number of *x*-intercepts
(2,-3)	upward		
(5,0)	downward		
(4,-2)	downward		

General Outcome: Develop algebraic and graphical reasoning through the study of relations.

Suggested Assessment Strategies

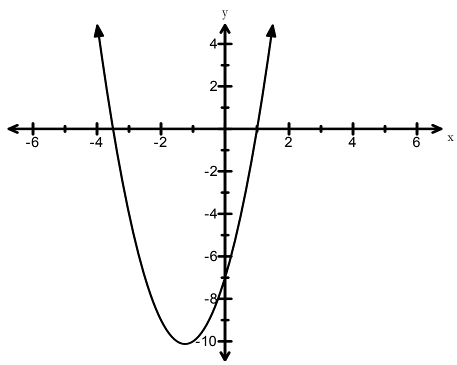
Journal

- Ask students to write a paragraph explaining the relationship between the roots of a quadratic equation, the zeros of a quadratic function and the *x*-intercepts of the graph of a quadratic function.

(RF2.3)

Paper and Pencil

- Ask students to answer the following:
  (i) Find the zeros of $f(x) = 2x^2 + 5x - 7$.
  (ii) Identify the *x*-intercepts of the graph.



  (iii) Find the roots of $2x^2 + 5x - 7 = 0$.
  (iv) What do you notice about the answers to the above questions?

(RF2.3)

- Ask students if the following statements are true or false. If a statement is false provide a counterexample to illustrate why the statement is false.
  (i) Some quadratic equations have no real solutions.
  (ii) A quadratic equation will always cross the *x*-axis in two distinct places.
  (iii) The graph of $y = x^2 + 2x + 2$ intersects the *x*-axis twice.

(RF2.3)

Performance

- Ask students to participate in the game *Card Flip*. Using a pack of 3 x 5 cards, place a quadratic function in vertex form on one side, with the number of *x*-intercepts on the other. Show the student the function and ask them to determine the number of *x*-intercepts on its graph. Flip the card to reveal the answer to the student.

(RF2.4)

Resources/Notes

Authorized Resource

Principles of Mathematics 11

7.1: Solving Quadratic Equations by Graphing

SB: pp.396-404
TR: pp.373-381

Relations and Functions

Outcomes

Students will be expected to

RF2 Continued ...

Achievement Indicators:

RF2.4 *Continued*

Elaborations—Strategies for Learning and Teaching

Students should be provided with enough examples to develop an understanding that the graph of a quadratic function may have zero, one or two x-intercepts depending on the location of the vertex and possibly the direction of opening. Ensure students are aware that the direction of the opening does not influence the number of x-intercepts when the vertex is located on the x-axis.

Ask students to participate in the following activity to check their understanding of quadratics and the possible number of x-intercepts. Teachers can distribute cards, where the coordinates of the vertex and direction of opening are placed on one side of the card and the number of x-intercepts is placed on the other side. In groups, ask students to look at the information and predict the number of x-intercepts. Flip the card to reveal the answer.

RF2.5 *Determine, by factoring, the roots of a quadratic equation, and verify by substitution.*

In Mathematics 1201, students factored quadratic polynomials of the forms $x^2 + bx + c$ and $ax^2 + bx + c$ where $a \neq 0$ (AN5). Students have also been exposed to special cases such as common factors, perfect square trinomials and differences of squares. A review of these factoring techniques may be necessary before solving quadratic equations by factoring.

Once a quadratic equation has been factored, students will use the zero product property to determine the roots. For example, when students factor $5x^2 + 14x - 3 = 0$ either $5x - 1 = 0$ or $x + 3 = 0$. Remind students to check their solutions by substituting the value of each root into the original equation and verify that the value makes the equation true.

Provide students with examples where the quadratic equation is missing the b or c value. When solving $x^2 - 5x = 0$, for example, students may factor $x(x - 5) = 0$ and then solve by dividing both sides of the equation by x, resulting in only one solution. Ask students if this is correct and whether a root has been eliminated. Encourage students to check the reasonableness of their answers using the zero product property.

Students should also be exposed to quadratic equations where $b = 0$. Consider equations similar to $x^2 - 4 = 0$. Students can use the method of solving a difference of squares but they should also be encouraged to use the square root property. This is an efficient method when students are solving equations similar to $x^2 - 8 = 0$, since the equation does not contain a perfect square.

General Outcome: Develop algebraic and graphical reasoning through the study of relations.

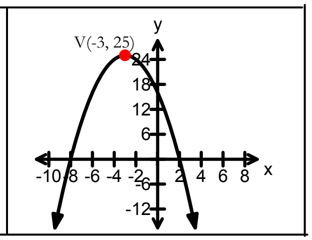
Suggested Assessment Strategies

Performance

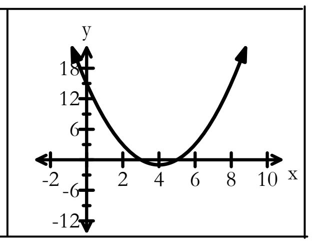
- Students could play the following game:

  Matching the Dominos: Give each pair of students a set of Domino Cards similar to those shown below. A set consists of 10 cards. The aim is for students to produce a closed loop of dominos, with the last two cards connecting the equation to its appropriate graph. Some cards will be blank which require students to complete the cards before they can place them in the loop.

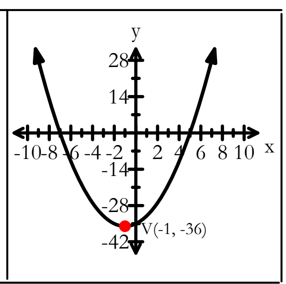
$y = x^2 - 8x + 15$



$y = x^2 + 2x - 35$



$y = -x^2 - 6x + 16$



(RF2.4)

Paper and Pencil

- The equation of the axis of symmetry of a quadratic function is $x = 0$ and one of the x-intercepts is -4. Ask students to determine the other x-intercept. They should explain using a diagram.

(RF2.4)

Resources/Notes

Authorized Resource

Principles of Mathematics 11

7.1: Solving Quadratic Equations by Graphing

SB: pp.396-404
TR: pp.373-381

7.2: Solving Quadratic Equations by Factoring

SB: pp.405-413
TR: pp.382-388

Relations and Functions

Outcomes

Students will be expected to

RF2 Continued ...

Achievement Indicators:

RF2.5 *Continued*

Elaborations—Strategies for Learning and Teaching

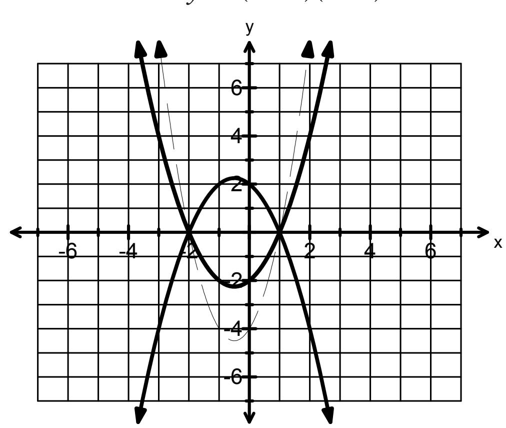
A quadratic equation may be given in a format which requires students to simplify before they are able to determine the roots. For example the equation $2x^2 + 2x - 12 = 3(x - 2)$ would require students to rewrite the equation in standard form, $2x^2 - x - 6 = 0$, before determining the roots. Students have had exposure to rearranging equations in the previous grades.

Reinforce the reasons why they are finding the roots of an equation and what the roots mean in the context of a problem. For example, the roots of an equation can help students find a horizontal distance. When a frog jumps into a body of water, for example, the height is zero before the frog jumps and when it hits the water. The difference between the two roots, the jumping point and the landing point, therefore, represents the distance the frog has travelled.

RF2.2 *Express a quadratic equation in factored form, given the zeros of the corresponding quadratic function or the x-intercepts of the graph of the function.*

In the previous unit, students were exposed to situations where they had to determine a unique parabola given the *x*-intercepts and another point. They will now write the quadratic equation based only on the *x*-intercepts. This would be a good opportunity to reinforce to students that manipulating the parameter *a* does not influence the value of the *x*-intercepts. Therefore, multiple quadratic equations exist. Students should analyze graphs to illustrate this concept. Consider the following equations resulting in *x*-intercepts -2 and 1.

$y = x^2 + x - 2$ $\quad y = (x + 2)(x - 1)$
$y = 2x^2 + 2x - 4$ $\quad y = 2(x + 2)(x - 1)$
$y = -x^2 - x + 2$ $\quad y = -(x + 2)(x - 1)$



In Mathematics 1201, students were exposed to the double distributive property (AN4). If students are asked to rewrite the factored form of the quadratic equation in standard form using the double distributive property, this will be a natural progression.

General Outcome: Develop algebraic and graphical reasoning through the study of relations.

Suggested Assessment Strategies

Performance

- Two sets of different coloured cards are required for this activity. One set will contain quadratic equations and the other set will have their corresponding solutions. Ask students to lay out the cards and match the equation card with its corresponding solution card.

$x^2 + 9x + 8 = 0$	$x = -1,\ x = 4$	$x = 3,\ x = -1$	$x^2 - 4 = 0$
$x = -2, x = 2$	$x^2 - 7x = 0$	$x^2 + 5x + 6 = 0$	$x = -6, x = 1$
$x = 0,\ x = 7$	$x^2 - 3 = 2x$	$x^2 - 3x - 4 = 0$	$x = -1,\ x = -8$
$x^2 - 6x + 8 = 0$	$x = -3, x = -2$	$x = 2,\ x = 4$	$x^2 + 5x + 6 = 12$

(RF2.5)

- Ask students to participate in the following game:
  Three in a Row: Students are given a 5 × 5 game board which consists of 25 squares. Each square will have a quadratic equation written on it. Ask them to play in pairs and take turns solving each quadratic equation with the aim to be the first person to make three in a row.

(RF2.5)

Journal

- Ask students to create an example of a quadratic function that cannot be solved by factoring. Explain why it cannot be factored.

(RF2.5)

Paper and Pencil

- Ask students to answer the following:
  (i) Write two different quadratic equations in standard form having roots -3 and 3.
  (RF2.2)
  (ii) If 1 is a solution to $ax^2 + bx + c = 0$, state the other factor.
  (RF2.2)
  (iii) Graph $y = a(x - 2)(x - 8)$ for $a = 1, 2, 3$ and 4.
  (a) What do these parabolas have in common?
  (b) Does the same property hold when a is negative? Explain.
  (RF2.2)

Resources/Notes

Authorized Resource

Principles of Mathematics 11

7.2: Solving Quadratic Equations by Factoring

SB: pp.405-413
TR: pp.382-388

Relations and Functions

Outcomes

Students will be expected to

RF2 Continued ...

Achievement Indicator:

RF2.6 *Determine, using the quadratic formula, the roots of a quadratic equation.*

Elaborations—Strategies for Learning and Teaching

Thus far, students have been exposed to solving factorable quadratic equations. The introduction of the quadratic formula allows students to explore situations where quadratic equations are not factorable.

The quadratic formula is used to find the roots of a quadratic equation in standard form, $ax^2 + bx + c = 0$, $a \neq 0$, where $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.

Since solving a quadratic equation using the method of completing the square is not an outcome in this course, students will not be expected to derive the quadratic formula. It is the expectation that students apply the formula only.

Students will be exposed to situations where the radicand in the quadratic formula simplifies to a negative number. This will be students' first exposure to square roots of negative numbers. It is not the intention of this course to introduce students to the imaginary number system but they need to be aware that there is no real solution to the square root of a negative number.

Some common errors occur when simplifying the quadratic formula. Students may:

- Apply the quadratic formula without ensuring the equation is written in standard form.
- Use $x = -b \pm \frac{\sqrt{b^2 - 4ac}}{2a}$ rather than the correct form of the quadratic formula.
- Incorrectly substitute the value for b and/or c back into the equation. When solving $x^2 - 5 = 0$, for example, students often substitute $b = 1$ rather than $b = 0$.
- Incorrectly produce two possible common errors if the b-value is negative.

  (i) If $b = -2$ then $-b = -(-2) = -2$

  (ii) If $b = -2$, then $b^2 = -2^2 = -4$

- Incorrectly simplify when applying the quadratic formula.

  (i) $\frac{2 \pm 4\sqrt{5}}{2} = \pm 2\sqrt{5}$

  (ii) $\frac{8 \pm \sqrt{5}}{2} = 4 \pm \sqrt{5}$

- Do not recognize that the ± results in two solutions. Suggest that students work through the solutions separately, showing calculations for both the positive solution and the negative solution.

General Outcome: Develop algebraic and graphical reasoning through the study of relations.

Suggested Assessment Strategies

Performance

- Ask students to participate in the game *Quadratic Tic-Tac-Toe*. Students should work in pairs for this activity. Each group is given a deck of cards, each with a quadratic equation on it. Students lay the cards out in a square grid (3 x 3). Students take turns flipping over a card and solving the equation using the quadratic formula. The first player with 3 in a row wins.

  (RF2.6)

- In groups of two, ask students to participate in the activity *Pass the Problem.* Distribute a problem to each pair that involves solving a quadratic equation using the quadratic formula. Ask one student to write the first line of the solution and then pass it to the second student. The second student will verify the workings and check for errors. If there is an error present, ask students to discuss the error and why it occurred. The student will then write the second line of the solution and pass it their partner. This process continues until the solution is complete.

  (RF2.6)

Paper and Pencil

- Ask students to solve the following equations:

  (i) $16x^2 + 8x = -5$

  (ii) $2x - 3 = 14x^2$

  (RF2.6)

Observation

- Ask students to work in pairs for this activity. Each group is given a deck of cards with various *a*, *b* and *c* values on them (each on a different colour paper). Students take turns selecting one card from each pile, building the quadratic equation that corresponds to their cards, and solving the equation using the quadratic formula. If they are correct, they get a point. If they are incorrect, students must work together to identify the error(s) made. The player with the most points at the end wins. Students may create a quadratic equation that leads to a negative radicand in the quadratic formula. This would be an opportunity for teachers to ask the following questions to the group:

  (i) What does this tell you about the number of *x*-intercepts?

  (ii) Could you change the parameter *a* so that *x*-intercepts exist?

  (RF2.6)

Resources/Notes

Authorized Resource

Principles of Mathematics 11

7.3: Solving Quadratic Equations Using the Quadratic Formula

SB: pp.414-424
TR: pp.389-398

Web Link

www.k12pl.nl.ca/seniorhigh/introduction/math2201/classroomclips.html

The **Quadratic Formula** clip demonstrates students taking turns solving a quadratic equation using the quadratic formula.

Relations and Functions

Outcomes

Students will be expected to

RF2 Continued ...

Achievement Indicator:

RF2.6 *Continued*

Elaborations—Strategies for Learning and Teaching

Quadratic equations can be solved using different methods. Factoring can only be used when the equation itself is factorable, but the quadratic formula can be used for all quadratic equations. When students use the quadratic formula to find roots of a quadratic equation, they will be exposed to trying to find the square roots of perfect squares, non-perfect squares and negative numbers.

It is important for students to distinguish between exact and approximate solutions. Consider the following equations and ask students to solve each of the equations using the quadratic formula.

A: $x^2 + 2x - 8 = 0$ B: $x^2 + x - 4 = 0$

$x = \frac{-2 \pm \sqrt{36}}{2}$ $x = \frac{-1 \pm \sqrt{17}}{2}$

Use the following questions to initiate a discussion around the square root component, $b^2 - 4ac$:

- What is the value under the square root in equation A? Is it a perfect square? How many roots exist and are they exact or approximate?
- What is the value under the square root in equation B? Is it a perfect square? How many roots exist and are they exact or approximate?
- What values of $b^2 - 4ac$ could lead to approximate answers? What values of $b^2 - 4ac$ could lead to exact answers?
- Which equation can also be solved by factoring?
- What connection can be made between an equation that is factorable and the value of $b^2 - 4ac$?

Although discriminant is not part of this course, it might be beneficial to discuss with students the conditions for a, b and c that are necessary for the quadratic formula to result in two real roots, one real root and no solution.

General Outcome: Develop algebraic and graphical reasoning through the study of relations.

Suggested Assessment Strategies

Performance

- Ask students to participate in the game *Chutes and Ladders*. Divide students into groups of four. Each group will be given a game board, 1 die, 1 pack of equation cards, and 4 game pieces to move along the board. Player one draws a card. Roll the die. If you roll a 1, then solve the quadratic equation by graphing. If you roll a 2 or a 3, then solve the quadratic equation by factoring. If you roll a 4 or a 5, then solve your quadratic equation by using the quadratic formula. If you roll a 6, solve it by a method of your choice. If you solve your equation correctly, then you move the number of spaces on the board that corresponds to your roll of the die. If you answer the question incorrectly, then the person to your left has the opportunity to answer your question and move those spaces. The first player to reach the end of the board first wins the game.

(RF2.1, RF2.5, RF2.6)

- Ask students to work in pairs for this activity. Provide students with a set of cards, each having a quadratic equation on it. Players take turns drawing an equation from the container. You may only choose one equation per turn. Do not return the equation to the container after your turn. Solve the quadratic equation you have been given. You receive 0 points if you choose an equation that has no real solution, 1 point for an equation that has 1 solution, and 2 points for an equation having 2 solutions. Keep a running total of your points. The player with the most points after the last equation is drawn from the container is the winner.

(RF2.1, RF2.5, RF2.6)

Presentation

- Ask students to create a poster/brochure that highlights the various methods they can use to solve a quadratic equation; the procedures involved, and an example of each.

(RF2.1, RF2.5, RF2.6)

Journal

- Describe the conditions for a, b, and c in $ax^2 + bx + c = 0$ that are necessary for the quadratic formula to result in only one possible root.

(RF2.6)

Resources/Notes

Authorized Resource

Principles of Mathematics 11

7.3: Solving Quadratic Equations Using the Quadratic Formula

SB: pp.414-424
TR: pp.389-398

Relations and Functions

Outcomes

Students will be expected to

RF2 Continued ...

Achievement Indicator:

RF2.7 *Solve a contextual problem by modelling a situation with a quadratic equation and solving the equation.*

Elaborations—Strategies for Learning and Teaching

Quadratic equations can be used to model a variety of situations such as projectile motion and geometry-based word problems. Students should be exposed to examples which require them to model the problem using a quadratic equation, solve the equation and interpret the solution.

Consider the following example:

- A rectangular lawn measuring 8 m by 4 m is surrounded by a flower bed of uniform width. The combined area of the lawn and flower bed is 165 m^2. What is the width of the flower bed?

It is important for students to recognize that the context of the problem dictates inadmissible roots. Discuss with students different scenarios that produce inadmissible roots. For example, time, height and length, would not make sense if they have a negative numerical value. However, temperature would make sense to have both negative and positive solutions.

When modelling situations for students, emphasize that restrictions sometimes need to be placed on the independent variable of the function. If a solution does not lie in the restricted domain, then it is not a solution to the problem. The following is an example with a restricted domain:

- A baseball is thrown from an initial height of 3 m and reaches a maximum height of 8 m, 2 seconds after it is thrown. At what time does the ball hit the ground?

In the above example the quadratic equation only models the path of the ball from the time it leaves the throwers' hand to the time it makes first contact with the ground. This quadratic equation yields two possible solutions, one of which is negative. This implies that it occurred before the ball was thrown. Therefore the restriction on the domain causes the negative solution to be inadmissible and only the positive solution is accepted.

General Outcome: Develop algebraic and graphical reasoning through the study of relations.

Suggested Assessment Strategies

Paper and Pencil

- A ball is thrown from a building at an initial height of 11 metres and reaches a maximum height of 36 metres, 5 seconds after it is thrown. Ask students to do the following:
  (i) Write a quadratic equation which models this situation.
  (ii) Three targets are placed at different locations on the ground. One is at (10,0), another at (11,0) and a final target is placed at (12, 0). Which target does the ball hit? Explain how you arrived at your answer.

(RF2.7)

- Ask students to find two consecutive whole numbers such that the sum of their squares is 265.

(RF2.7)

- A diver's path when diving off a platform is given by $d = -5t^2 + 10t + 20$, where d is the distance above the water (in feet) and t is the time from the beginning of the dive (in seconds).
  (i) How high is the diving platform?
  (ii) After how many seconds is the diver 25 feet above the water?
  (iii) When does the diver enter the water?

(RF2.7)

- Ask students to choose a quadratic word problem from the class notes or group work stations and use it as a guide to create their own word problem. Remember to have them include their solutions on a separate sheet and give their problem to another student to solve.

(RF2.7)

Presentation

- Ask students to divide into small groups and research one of the following:
  (i) The history of quadratic equations.
  (ii) The origins of the terms used to describe quadratic equations (ie. vertex, axis of symmetry, etc.)
  (iii) How quadratic equations are used in application type problems.

(RF2.7)

Resources/Notes

Authorized Resource

Principles of Mathematics 11

7.4: Solving Problems Using Quadratic Equations

SB: pp.425-432
TR: pp.399-407

Proportional Reasoning

Suggested Time: 10 Hours

Unit Overview

Focus and Context

In this unit, students will represent, interpret and compare rates and unit rates. They will also explore the relationship between the slope of a graph and a rate. Students will connect rates to contextual problems. Unit rates can be calculated to help them make decisions, such as, which store has the lower cost.

Students will be exposed to scale factors for scale diagrams of 2-D shapes. They will then focus on the relationship bewtween scale factor and area.

They will extend their work with scale factors and scale diagrams for 2-D shapes to scale factors and scale models for 3-D shapes. They will focus on scale factors related to surface area and volume.

Outcomes Framework

GCO

Develop spatial sense and proportional reasoning.

SCO M1

Solve problems that involve the application of rates.

SCO M2

Solve problems that involve scale diagrams, using proportional reasoning.

SCO M3

Demonstrate an understanding of the relationships among scale factors, areas, surface areas and volumes of similar 2-D shapes and 3-D objects.

Process Standards Key

[**C**] Communication	[**PS**] Problem Solving
[**CN**] Connections	[**R**] Reasoning
[**ME**] Mental Mathematics and Estimation	[**T**] Technology
	[**V**] Visualization

SCO Continuum

Mathematics 1201	Mathematics 2201	Mathematics 3201
Measurement	Measurement	
M2 Apply proportional reasoning to problems that involve conversions between SI and imperial units of measure. [C, ME, PS]	M1 Solve problems that involve the application of rates. [CN, PS, R] M2 Solve problems that involve scale diagrams, using proportional reasoning. [CN, PS, R, V] M3 Demonstrate an understanding of the relationships among scale factors, areas, surface areas and volumes of similar 2-D shapes and 3-D objects. [C, CN, PS, R, V]	

Measurement

Outcomes

Students will be expected to

M1 Solve problems that involve the application of rates.

[CN, PS, R]

Achievement Indicators:

M1.1 *Interpret rates in a given context, such as the arts, commerce, the environment, medicine or recreation.*

M1.2 *Determine and compare rates and unit rates.*

M1.3 *Make and justify a decision, using rates.*

Elaborations—Strategies for Learning and Teaching

In Grade 8, students explored the difference between a rate and a unit rate (8N5). In Grade 9, they used the concept of scale factor to create enlargements and reductions of 2-D shapes (9SS4). In Mathematics 1201, students made the connection between the concept of slope as a measure of rate of change (RF3). In this unit, students will represent a rate in different ways, determine a unit rate, and will then use unit rates to solve problems and make decisions.

Students have previously been introduced to rate as a comparison between two things with different units. Invite students to talk about comparisons they make in their lives. Suggestions may include fuel consumption, speed, prices in supermarkets, monthly fees of a fitness centre, etc.

Students will be expected to differentiate between rate and unit rate. Ask students to suggest some common unit rates. They may respond with kilometers per hour, cost per item, earnings per week, etc. In each case, it is important for them to recognize that the first quantity is related to one unit of the second quantity. Ask students what the benefits of using unit rates are. Pose questions such as:

- Would the cost of one unit make it easier to find the cost of multiple units?
- How can you compare the unit rates to make a decision?

Students will be exposed to problems, such as the following, where they compare rates expressed in different units:

In the month of May, Jessica sent 4216 text messages. During a one-week period, Patrick sent 1008 texts. Who has the higher text rate per day?

Emphasize that problems involving rates are compared using proportional reasoning and by calculating unit rates.

This would be an opportunity to make a connection between unit rates and unit pricing. The unit price is the amount a consumer pays for each unit of the product they are buying. Students can use unit pricing to compare prices of items and to determine which price is the better deal. Pose a scenario such as: Paper towels are sold in a 2-roll package for $2.49 and a 12-roll package for $12.99.

Students should answer the following:

- What package has the lower unit price?
- How much would you save by buying a 12-roll package rather than six 2-roll packages?
- When deciding which package size is the better buy for you, what should you consider in addition to unit price?

General Outcome: Develop spatial sense and proportional reasoning.

Suggested Assessment Strategies

Paper and Pencil

- Ask students to answer the following:
  - (i) Sean buys a package of 15 pencils for $4.50 at Walmart. Angela buys a box of 50 pencils at Costco for $14.00. Which is the better buy?

    (M1.1, M1.2, M1.3)
  - (ii) Each of your finger nails grows at about 0.05 cm/week. Each of your toenails grows at about 0.65 cm/year. Do your toenails or finger nails grow faster?

    (M1.1, M1.2, M1.3)

Observation

- In groups of two, ask one student to state a rate while the other student states it as a unit rate. Students can then present their answers to the rest of the class.

  (M1.2)

Performance

- Ask students to work in centres with each centre containing similar items in different sizes with prices given (e.g., soup, pop, dog food, shampoo, etc.). In their journals, students discuss the item they would buy and why they feel it is the better buy.

  (M1.1, M1.2, M1.3)

Resources/Notes

Authorized Resource

Principles of Mathematics 11

8.1 Comparing and Interpreting Rates

Student Book(SB):pp.444-453
Teacher Resource(TR):pp.435-442

Measurement

Outcomes

Students will be expected to

M1 Continued ...

Achievement Indicators:

M1.1, M1.2, M1.3 *Continued*

M1.4 *Draw a graph to represent a rate.*

M1.5 *Explain, using examples, the relationship between the slope of a graph and a rate.*

Elaborations—Strategies for Learning and Teaching

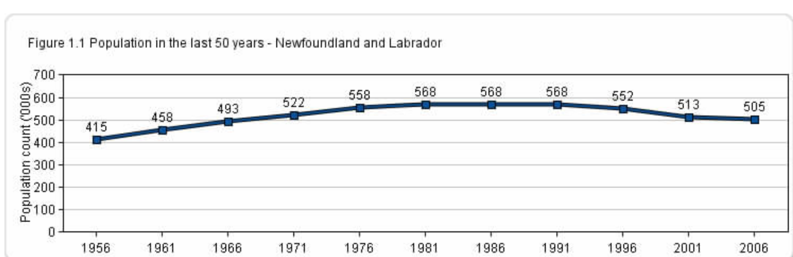
In stores that provide unit pricing, students can look for special shelf tags below the product. The labels are often easy to overlook. They may be the most helpful items in the store when it comes to getting the best buy. It is important for students to recognize, however, that buying large quantities of items that have a cheaper unit price is not helpful if some of the product is wasted because it was not fully used or has expired.

Engage students in a group activity involving comparative shopping. Ask students to collect flyers to compare various products and determine the better buy based on the unit prices. After doing some comparison, they should decide what store they would recommend and why. They should also consider other factors that would influence their decision to purchase items.

In Mathematics 1201, students were introduced to the concept of slope and how it measures the rate of change in the dependent variable as the independent variable changes (RF3). The concept of slope is important, for example, in economics. Economists often look at how things change and how one item changes in response to a change in another item. They may observe, for example, how demand changes when price changes.

Students will be expected to draw and interpret graphs which illustrate various rates, such as gas rates, speed, etc. Consider the following example:

Using the graph, discuss the population change of Newfoundland from 1956 to 2006.



www.statcan.gc.ca

Pose the following questions to promote student discussion:

- What does a positive slope represent? A negative slope?
- What does the slope of a horizontal line represent?
- What does the steepness of the slope represent?

General Outcome: Develop spatial sense and proportional reasoning.

Suggested Assessment Strategies

Paper and Pencil

- Ask students to answer the following questions:
  (i) The following table represents the average cost per litre of regular gasoline in Newfoundland and Labrador for the first six months of 2011.

Month	January	February	March	April	May	June
Cost/ Volume ($/L)	121.2	123.6	131.4	139.2	133.1	131.9

  Using a graph, determine between which months the cost per litre of gas decreased the least amount. Give reasons why you think this occurred.

(M1.4, M1.5)

  (ii) The average monthly temperature in Corner Brook, NL, in °C , is recorded in the chart below. The sampling period for this data covers 30 years.

Month	Average High
January	-2.5
February	-3.2
March	1.1
April	6.4
May	12.1
June	17.7
July	21.8
August	21.1
September	16.7
October	10.5
November	4.0
December	0.2

  Using a graph, identify between which two months the rate of change is the greatest. Between which two months is the rate of change the least?

(M1.4, M1.5)

Resources/Notes

Authorized Resource

Principles of Mathematics 11

8.1 Comparing and Interpreting Rates

SB: pp. 444-453
TR: pp. 435-442

Measurement

Outcomes

Students will be expected to

M1 Continued ...

Achievement Indicators:

M1.6 *Describe a context for a given rate or unit rate.*

M1.7 *Identify and explain factors that influence a rate in a given context.*

M1.8 *Solve a contextual problem that involves rates or unit rates.*

M1.9 *Solve a rate problem that requires the isolation of a variable.*

Elaborations—Strategies for Learning and Teaching

The focus here is to describe a situation in which a given rate might be used and is most useful. A situation to describe the rate 5¢/min, for example, could be a long distance phone call within Canada.

Students will also be expected to determine the reasonableness of rates. If a student was having a party and trying to determine the number of slices of pizza per person, would it matter if the party was for a 3 year old as opposed to a teenager?

Ask students to answer questions such as the following if they were planning a road trip to Las Vegas:

- Is it reasonable to discuss a road trip to Las Vegas in terms of m/s?
- What rate(s) could be used to describe this road trip?
- What factors might affect your rate of speed on this trip?
- What factors might affect your fuel consumption?

Providing students the opportunity to work with rates in real-life scenarios should reinforce the students' understanding of rates, their usefulness, and their reasonableness.

Students should be provided with several examples using various units and rates. Teachers should take this opportunity to work with problems involving rates that can be solved using equivalent ratios, proportional reasoning and/or unit analysis. Examples may include the following:

- During a Terry Fox Run, student volunteers distribute 250 mL cups of water to participants as they cross the finish line. Each volunteer has a cooler that can hold 64 L of water. How many cups of water can each volunteer dispense?
- Loose-leaf paper costs $1.49 for 200 sheets or $3.49 for 500 sheets. What is the least you can pay for 100 sheets? 1600 sheets?

General Outcome: Develop spatial sense and proportional reasoning.

Suggested Assessment Strategies

Paper and Pencil

- Ask students to answer the following questions:
  - (i) Betty earns $463.25 in 5 weeks. How much will she earn in 2 years?

    (M1.6, M1.8, M1.9)
  - (ii) A 12-bottle case of motor oil costs $41.88. A mechanic needs to order 268 bottles of motor oil. If he can only order by the case, how much money does he spend?

    (M1.6, M1.8, M1.9)

Journal

- Ask students to list reasons why the unit price is often lower for a larger size of a product than for the smaller size of the same product.

  (M1.6, M1.8, M1.9)

Performance

- Ask students to collect store flyers that advertise food products. With a classmate or a group, compare prices of 4 different stores. Where product sizes differ, calculate the best value.

  (M1.6, M1.8, M1.9)

Resources/Notes

Authorized Resource

Principles of Mathematics 11

8.2 Solving Problems That Involve Rates

SB: pp. 454-462
TR: pp. 443-451

Measurement

Outcomes

Students will be expected to

M2 Solve problems that involve scale diagrams, using proportional reasoning.

[CN, PS, R, V]

Achievement Indicators:

M2.1 *Explain, using examples, how scale diagrams are used to model a 2-D shape.*

M2.2 *Determine, using proportional reasoning, an unknown dimension of a 2-D shape, given a scale diagram or model.*

M2.3 *Determine, using proportional reasoning, the scale factor, given one dimension of a 2-D shape, and its representation.*

Elaborations—Strategies for Learning and Teaching

In Grade 9, students were introduced to scale factors and scale diagrams (9SS4). Students explored the concepts of enlargements and reductions. They also determined the scale factor given the scaled diagram of two dimensional images, and used a scale factor to create an image from its original figure. In this unit, students will use scale diagrams involving 2-D shapes before moving on to 3-D objects.

Although students were exposed to scale factors and scale diagrams of 2-D shapes in Grade 9, a review of these concepts is important. This is the first time that a variable, such as *k*, will be used to represent the scale factor, $k = \frac{\text{diagram measurement}}{\text{actual measurement}}$.

Using scale factors and measurements, students should be able to determine the dimensions of a reduced or enlarged object. Consider the following example:

A dinosaur model has a scale of 1:12. If the head of the dinosaur model is 8 cm in length, how long is the head of the real dinosaur? Use questions such as the following to promote student discussion:

- What is the scale factor?
- What does this value mean?
- Will the value of the scale factor result in an enlargement or reduction? Explain.
- How did you determine the length of the head of the dinosaur?

Students should recognize that for each 1 cm the model measures, the corresponding part of the dinosaur measures 12 cm. They may set up and solve a proportion equation or they may simply recognize the dinosaur is 12 times the length of the model dinosaur.

Students should also be able to determine scale factors from a variety of sources, such as models and scale diagrams, by using corresponding lengths. As students work through different examples, they should recognize that the scale factor for an enlargement is greater than one and the scale factor of a reduction is between 0 and 1.

When working with problems, students should be exposed to scale factors in a variety of forms including decimals, fractions and percentages. Review of the conversion of different units in both the metric and imperial systems may be necessary. It is also important to emphasize the meaning of units so students better understand the concept of scale and can gain a visual appreciation of the object that is being reduced or enlarged.

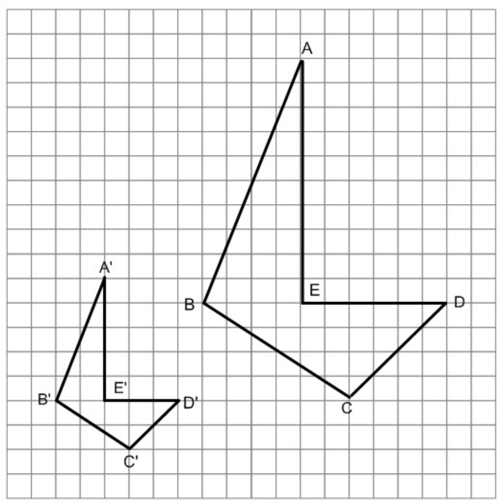
General Outcome: Develop spatial sense and proportional reasoning.

Suggested Assessment Strategies

Paper and Pencil

- Ask students to answer the following questions:
  - (i) A circle has been transformed so that its image radius is 14 cm. If the scale factor is 0.4, what is the radius of the original circle?

    (M2.2, M2.3)
  - (ii) Determine the scale factor from the diagram:



    (M2.2, M2.3)
  - (iii) ΔABC is similar to ΔSTU. If the sides of ΔABC measure 5.2, 3.8, and 6.4 respectively, and the scale factor is 2:3, what is the perimeter of ΔSTU ?

    (M2.2, M2.3)

Resources/Notes

Authorized Resource

Principles of Mathematics 11

8.3 Scale Diagrams

SB: pp. 466-474
TR: pp. 452-458

Measurement

Outcomes

Students will be expected to

M2 Continued...

Achievement Indicators:

M2.4 *Draw, with or without technology, a scale diagram of a given 2-D shape, according to a specified scale factor (enlargement or reduction).*

M2.5 *Solve a contextual problem that involves a scale diagram.*

Elaborations—Strategies for Learning and Teaching

Examples of real-world applications, such as maps, sewing patterns, car models, and construction blueprints, would be an effective way to introduce this concept and capture students' interest. Students could be given a road map, the price of gasoline per litre and the miles per gallon value for a vehicle and asked to determine an estimate of the cost for a trip between two towns.

An appropriate activity would involve students designing a floor plan of their bedroom, kitchen or a room of their choice. They will have to consider what to include in their drawing, the scale they will use and the measurements needed. Students must also realize that it is important for the measurements to be realistic. If a blueprint is using a scale of 1 in. = 1 ft, for example, the doorway should be at least 3 in. wide on the drawing so that in real-life it would be 3 ft wide.

Consider asking the following questions to guide students with their design:

- How are the dimensions recorded on the diagram?
- How are the doors, windows, closets and walls represented?
- Is the scale indicated?
- Should a key be included to identify the symbols used in the drawing?
- Will furniture be included?

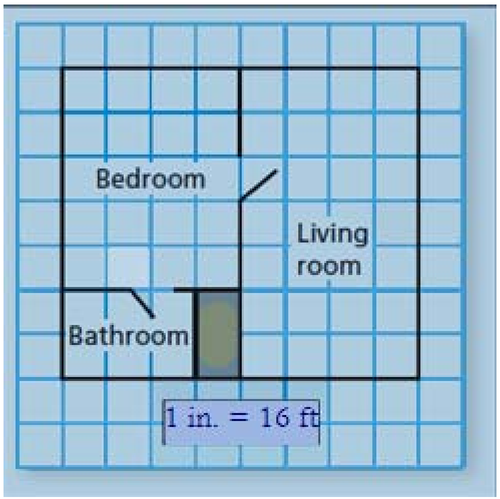
Completed floor plans can be posted around the classroom for students to see other examples.

General Outcome: Develop spatial sense and proportional reasoning.

Suggested Assessment Strategies

Paper and Pencil

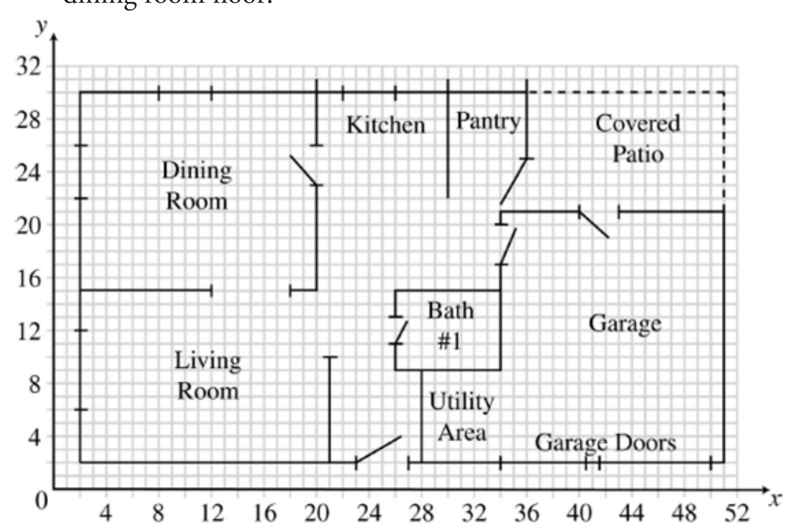
- In the following blueprint, each square has a side length of $\frac{1}{4}$ in. Ask students to answer the following questions:



(i) Ceramic tile costs $5 per square foot. How much would it cost to tile the bathroom?
(ii) Carpet costs $18 per square yard. How much would it cost to carpet the bedroom and living room?
(iii) Which has a higher unit cost, the tile or the carpet? Explain.

(M2.4, M2.5)

- Mrs. Lewis intends to have the dining room redone with wood flooring that comes in 1 foot wide squares. The scale is 1 unit = 1ft. A 12 pack of squares sells for $38.40 and single squares can be purchased for $3.70 each. Ask students to determine the cost, in dollars, for Mrs. Lewis to buy just enough squares to cover her dining room floor?



(M2.4, M2.5)

Resources/Notes

Authorized Resource

Principles of Mathematics 11

8.3 Scale Diagrams

SB: pp. 466-474
TR: pp. 452-458

Measurement

Outcomes

Students will be expected to

M3 Demonstrate an understanding of the relationships among scale factors, areas, surface areas and volumes of similar 2-D shapes and 3-D objects.

[C, CN, PS, R, V]

Achievement Indicators:

M3.1 *Explain, using examples, the effect of a change in the scale factor on the area of a 2-D shape.*

M3.2 *Determine the area of a 2-D shape, given the scale diagram, and justify the reasonableness of the result.*

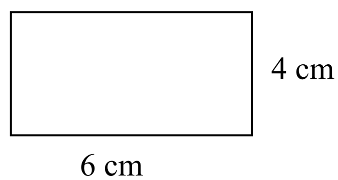
M3.3 *Solve a spatial problem that requires the manipulation of formulas.*

M3.4 *Solve a contextual problem that involves the relationships among scale factors, areas and volumes.*

Elaborations—Strategies for Learning and Teaching

Students will focus on the relationship between scale factor and area of similar 2-D shapes. Later in this unit, they will solve problems that involve scale factor, surface area and volume of 3-D objects.

Students will analyze how area is affected when the lengths of shapes are enlarged or reduced by a particular scale factor. This may be a good opportunity to review area formulas for shapes such as a parallelogram, triangle, rectangle and circle. The following rectangle could be used as an illustration.



Students will explore the relationship between scale factor and area. They should use different scale factors such as 2, 3, 0.5, etc. Consider the following questions for discussion:

- What is the area of the original rectangle?
- What are the dimensions and area of the resulting similar rectangles?
- What do you notice?

Students should observe that the resulting areas are not directly proportional to the lengths. When students double the sides of a rectangle, for example, the area does not just double, it quadruples.

It is important for them to recognize that the scale factor is applied to each dimension of the 2-D shape. As a result, the area will either increase or decrease by a factor of k^2. Therefore, $k^2 = \frac{\text{area of similar 2-D shape}}{\text{area of original shape}}$.

General Outcome: Develop spatial sense and proportional reasoning.

Suggested Assessment Strategies

Paper and Pencil

- Ask students to answer the following questions:
  (i) A 4 in. by 6 in. picture frame has dimensions that have been tripled. What is the area of the new frame?
  (M3.2, M3.4)

  (ii) Chad and Charlene painted a mural on the wall, measuring 12 ft by 8 ft using an overhead projector. If the original sketch had an area of 216 in.^{2}, what is the scale factor?
  (M3.2, M3.4)

Journal

- Ask students to find at least three examples of enlargements or reductions in real-world objects. The should estimate their scale factors.
  (M3.2, M3.4)

Resources/Notes

Authorized Resource

Principles of Mathematics 11

8.4 Scale Factors and Areas of 2-D Shapes

SB: pp. 475-482
TR: pp. 459-465

Measurement

Outcomes

Students will be expected to

M2 Continued...

Achievement Indicators:

M2.5 *Continued*

M2.6 *Explain, using examples, how scale diagrams are used to model a 3-D object.*

M2.7 *Determine, using proportional reasoning, the scale factor, given one dimension of a 3-D object, and its representation.*

M2.8 *Determine, using proportional reasoning, an unknown dimension of a 3-D object, given a scale diagram or model.*

Elaborations—Strategies for Learning and Teaching

Students will extend their work with scale factors and scale diagrams of 2-D shapes to scale factors and scale diagrams of 3-D objects. This will be their first exposure to relating scale factors to a 3-D object.

Students will use a scale factor to determine unknown measurements of similar 3-D objects. This would be a good opportunity to bring in a regular size cereal box and a similar jumbo size cereal box. Students can create nets of the boxes and record the measurements. Ask if the boxes are similar and why. Are the dimensions related by a scale factor? Students should recognize that corresponding measurements of the boxes (length, width and height) are proportional.

Students should also be able to use a given scale factor to determine the unknown dimensions of a 3-D object. If the dimensions of a scale drawing of a patio chair are 2 cm by 1.5 cm by 4 cm, for example, and a scale factor of 1:30 is applied, ask them to determine the actual dimensions of the patio chair.

It is important to utilize various everyday objects and apply a given scale factor when asking students to determine the dimensions of the actual object or its image.

General Outcome: Develop spatial sense and proportional reasoning.

Suggested Assessment Strategies

Paper and Pencil

- Ask students to answer the following questions:

  (i) During an Art class, students are projecting the image of a can of Carnation milk on the wall. The projector applies a scale factor of 250%. If the can has a diameter of 10 cm and a height of 12.5 cm, what are the dimensions of the image on the wall?

  (M2.7, M2.8)

  (ii) Tony drew a scale diagram of his new skateboard to show a friend. He used a scale factor of 0.4. The scaled diagram has dimensions 3.2 in. by 1.8 in. by 10.8 in. What are the dimensions of the skateboard?

  (M2.7, M2.8)

Resources/Notes

Authorized Resource

Principles of Mathematics 11

8.5 Similar Objects: Scale Models and Scale Diagrams

SB: pp. 483-493
TR: pp. 466-472

Measurement

Outcomes

Students will be expected to

M3 Continued...

Achievement Indicator:

M3.5 *Explain, using examples, the effect of a change in the scale factor on the surface area and volume of a 3-D object.*

Elaborations—Strategies for Learning and Teaching

Students will now investigate the relationship between the scale factor and the surface area of two similar 3-D objects, in addition to the relationship between the scale factor and the volume of two similar 3-D objects. This would be a good opportunity to use manipulatives such as linking cubes.

Ask students to measure the dimensions of 1 linking cube. The volume of such a cube is 1 cm^3 while its surface area is 6 cm^2. Ask them what will happen if every edge of the cube is doubled. Students should observe that the volume of the cube increases by a factor of 8 or 2^3, while the surface area only increases by a factor of 4 or 2^2.

Ask students what will happen to the surface area and the volume of the original cube if every edge of the original cube triples.

Encourage students to work in groups, build various cubes using pre-determined scale factors, find the surface area and volume, and make the connection to its scale factor. Information could be organized in a chart.

Scale factor	Length	Width	Height	Surface Area	Volume
1	1	1	1	6	1
2	2	2	2	24 (increases by a factor of 4)	8 (increases by a factor of 8)
3	3	3	3	54 (increases by a factor of 9)	27 (increases by a factor of 27)
4	4	4	4	96	64

Students should recognize that when the dimensions of similar 3-D objects are related by a scale factor k, their surface areas are related by k^2 and their volumes are related by k^3.

General Outcome: Develop spatial sense and proportional reasoning.

Suggested Assessment Strategies

Journal

- Ask students to help resolve an argument between three of their friends. David claims that when you enlarge every side of a cube n times, its volume also increases n times. Jane says that the volume of a cube increases 3n times, and Anne is convinced that the volume increases n^3 times. Ask students who they agree with and why.

(M3.5)

Resources/Notes

Authorized Resource

Principles of Mathematics 11

8.6 Scale Factors and 3-D Objects

SB: pp. 494-503
TR: pp. 473-483

Measurement

Outcomes

Students will be expected to

M3 Continued...

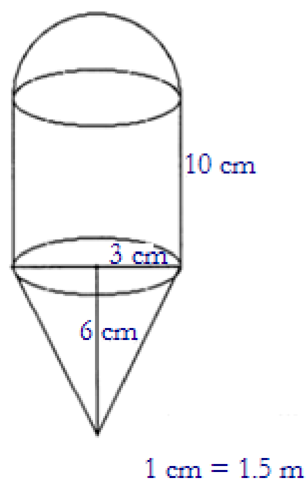
Achievement Indicator:

M3.6 *Determine the surface area and volume of a 3-D object, given the scale diagram, and justify the reasonableness.*

Elaborations—Strategies for Learning and Teaching

In Mathematics 1201, students solved problems that involved the surface area and volume of 3-D objects (M3). It would be beneficial to review the surface area and volume formulas of 3-D objects such as a rectangular prism, right cylinder, right cone, right pyramid and sphere.

Students can use the dimensions of a scale diagram of a 3-D object, as well as the scale factor, to determine the surface area and volume of the enlarged/reduced object. Consider the scaled down diagram of the following storage tank:



Guide students through the process using the following instructions and questions:

- Find the total surface area of the original tank.
- Find the total volume of the original tank.
- Was it necessary to determine the dimensions of the original drawing to answer the above questions?
- How would you determine the surface area and volume of the original if the scale diagram was not given?

General Outcome: Develop spatial sense and proportional reasoning.

Suggested Assessment Strategies

Paper and Pencil

- Ask students to answer the following questions:
  - (i) The surface area of a cone is 36 ft2. What is the surface area of its image if a scale factor of 1:4 is applied?

    (M3.6)
  - (ii) Find the volume of a cylinder if its image has a volume of 450 cm^3 and a scale factor of 2:3. Round your answer to the nearest cubic centimetre.

    (M3.6)
  - (iii) What is the scale factor of the following pairs of similar spheres?
    - (a) Volume of the original is 450 mm^3 and its image is 1518.75 mm^3.
    - (b) Surface area of the original is 248 in^2 and its image is 126.5306 in^2.

    (M3.6)

Resources/Notes

Authorized Resource

Principles of Mathematics 11

8.6 Scale Factors and 3-D Objects

SB: pp. 494-503
TR: pp. 473-483

REFERENCES

American Association for the Advancement of Science [AAAS–Benchmarks]. *Benchmarks for Science Literacy.* New York, NY: Oxford University Press, 1993.

Armstrong, Thomas. *7 Kinds of Smart: Identifying and Developing Your Many Intelligences.* New York, NY: Plume, 1993.

British Columbia Ministry of Education. *The Primary Program: A Framework for Teaching*. Victoria, BC: British Columbia Ministry of Education, 2000.

Caine, Renate Nummela and Geoffrey Caine. *Making Connections: Teaching and the Human Brain*. Alexandria, VA: Association for Supervision and Curriculum Development, 1991.

Canavan-McGrath, Cathy et al. *Principles of Mathematics 11*. Toronto, ON: Nelson Education Ltd., 2011

Faulkenberry, Eileen and Thomas Faulkenberry. "Transforming the Way We Teach Function Tranformation." *Mathematics Teacher* 104, 1 (August 2010), pp. 29-33.

Government of Newfoundland and Labrador. *Foundations for the Atlantic Canada Mathematics Curriculum.* Halifax, NS: Atlantic Provinces Education Foundation.

Government of Newfoundland and Labrador: Department of Education. *Mathematics Grade 10 Interim Edition Curriculum Guide 2011*. Retrieved March 2012 from the World Wide Web: http://www.ed.gov.nl.ca/edu/k12/curriculum/guides/mathematics/index.html.

Hope, Jack A. et al. *Mental Math in Junior High.* Palo Alto, CA: Dale Seymour Publications, 1988.

Hope, Jack A. et al. *Mental Math in the Primary Grades.* Palo Alto, CA: Dale Seymour Publications, 1988.
Keeley, Page and Cheryl Rose Tobey. *Mathematics Formative Assessment*. Thousand Oaks, California: Corwin, 2011.

Kelly, Brendan. et al. *Addison-Wesley Mathematics 12.* Don Mills, Ontario: Addison-Wesley Publishers Limited., 1991.

Knill, George. et. al. *Math Power 11.* Toronto: McGraw-Hill Ryerson Limited, 1999.

National Council of Teachers of Mathematics. *Computation, Calculators, and Common Sense: A Position of the National Council of Teachers of Mathematics.* May 2005. http://www.nctm.org/about/pdfs/position/computation.pdf (Accessed February 22, 2007).

Rubenstein, Rheta N. "Mental Mathematics beyond the Middle School: Why? What? How?" *Mathematics Teacher* 94, 6 (September 2001), pp. 442–446.

Shaw, J. M. and M. J. P. Cliatt. "Developing Measurement Sense." In P. R. Trafton (ed.), *New Directions for Elementary School Mathematics: 1989 Yearbook* (Reston, VA: National Council of Teachers of Mathematics, 1989), pp. 149–155.

Small, Marian. *Making Math Meaningful to Canadian Students, K-8.* Toronto, ON: Nelson Education, 2008.

Steen, L. A., ed. *On the Shoulders of Giants: New Approaches to Numeracy*. Washington, DC: Mathematical Sciences Education Board, National Research Council, 1990.

Van de Walle, John A. *Elementary and Middle School Mathematics: Teaching Developmemtally*. 4th ed. Boston, MA: Addison Wesley LAyman, INC., 2001.

Western and Northern Canadian Protocol for Collaboration in Basic Education (Kindergarten to Grade 12). *The Common Curriculum Framework for K–9 Mathematics: Western and Northern Canadian Protocol* – May 2006 and *The Common Curriculum Framework for Grades 10–12* – January 2008. Reproduced (and/or adapted) by permission. All rights reserved.