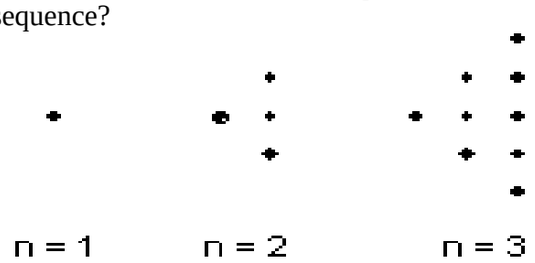
Math 2201 Unit 1 Test Name: _____

Part I: Multiple Choice. Place the correct answer on the space provided. (2 marks)

____ 1. The first three members of a sequence are shown. How many dots are in the fourth member of the sequence?



a. 30 b. 16 c. 14 d. 7

____ 2. State a counterexample to disprove the following conjecture: A hot air balloon is a device that floats in the air.
a. Hot air balloons are red.
b. Helium balloons also float.
c. A car is also a device.
d. Hot air is also warm.

Part II: Answer each question in the space provided. Show ALL workings to receive full marks!

3. Give one counterexample that shows the conjecture is false. **(1 mark)**
In a coordinate plane, if the *y*-coordinate of a point is positive, then the point is in the first quadrant.

4. Give a counterexample to the following conjecture. **(1 mark)**
All mammals cannot fly.

5. Give a counterexample to the following conjecture. **(1 mark)**
The sum, $2^n + 1$ where *n* is a natural number, is always a prime number.

Use inductive reasoning to find the next two numbers in each pattern. (2 mark)

6. 16, 18, 20, 22, __, __ **(2 mark)**

7. 2, 4, 8, 16, __, __ **(2 mark)**

From the given true statements, make a valid conclusion:

8. If there is no more milk, Rita will go to the store. **(1 mark)**
There is no more milk.

9. If the slipper fits, she is the one. **(1 mark)**
Cinderella fits in the slipper.

10. Use deductive reasoning to show that the difference of two even numbers is even. **(4 mark)**

11. Use deductive reasoning to prove the conjecture: The square of an odd integer is always an odd integer. (*Hint*: Represent the original integer as $2n + 1$) **(4 mark)**

Decide if the argument is valid or invalid. If the argument is valid, tell which rule of logic is used. If the argument is invalid, tell why.

12. If a figure is a quadrilateral, then it is a polygon. **(2 mark)**
I have drawn a figure that is a polygon.
Therefore, the figure I drew is a quadrilateral.

13. The following proof seems to show that 2 = 1. Examine this proof, and determine where the error in reasoning occurred. **(2 mark)**

Step 1: Let a = b

Step 2 : $a^2 = ab$ Multiply by a

Step 3: $a^2 - b^2 = ab - b^2$ Subtract b^2

Step 4: $(a - b)(a + b) = b(a - b)$ Factor

Step 5: $a + b = b$ Divide by $(a - b)$

Step 6: $b + b = b$ $a = b$

Step 7: $2b = b$ Simplify

Step 8: $2 = 1$ Divide by b

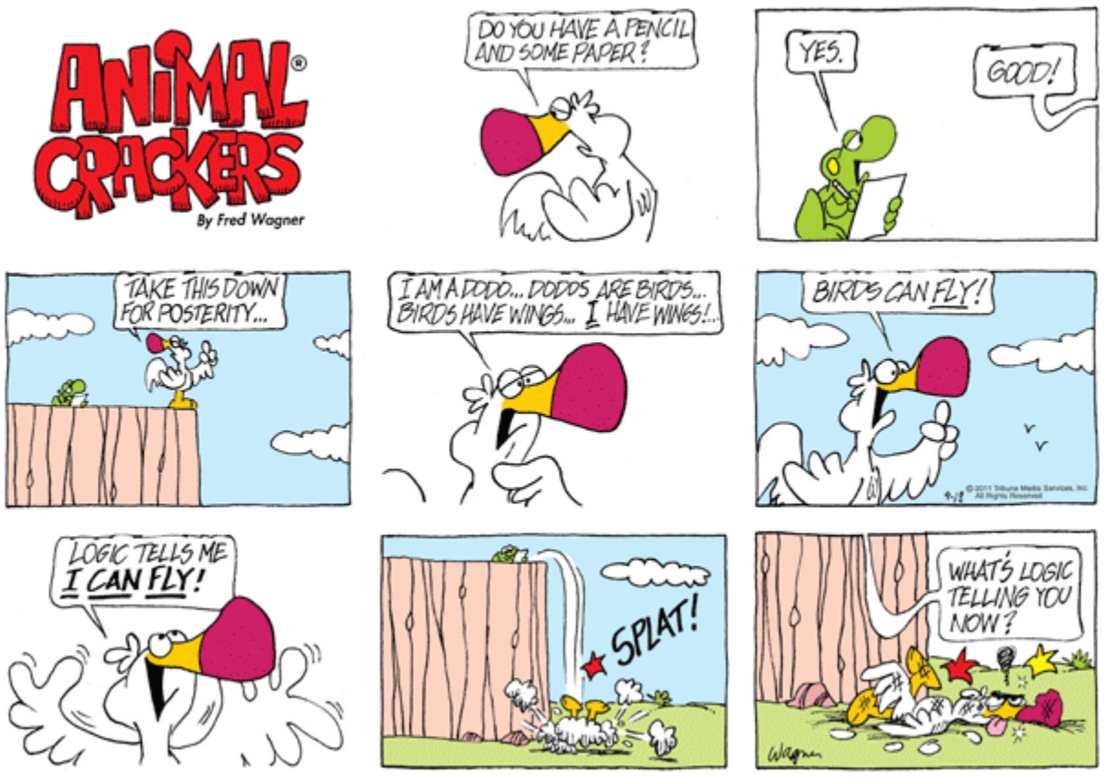
14. Use inductive reasoning to make a conjecture for the magic trick shown below. Then use deductive reasoning to prove your conjecture.

a) Inductive Reasoning **(2 points): Fill in case 1 and case 2**

	Case 1	*Case 2*	*General Case*
Step 1: Choose a number			
Step 2: Double the number			
Step 3: Add 6 to the result			
Step 4: Divide the sum by 2			
Step 5: Subtract 3 from the result			

b) Conjecture **(1 mark):**

c) Deductive Reasoning **(2 marks):** Use General case section in above chart. (Hint: Use n for the original number.)



Answer Section

1. ANS: B
2. ANS: B
3. ANS:
   Answers will vary.
   The point (–4, 5) is not in the first quadrant.
4. ANS:
   Answers will vary.
   For example,
   bats are mammals that can fly.
5. ANS:
   Answers will vary. For example,
   when $n = 3$, the expression gives 9.
6. ANS:
   24, 26
7. ANS:
   32, 64
8. ANS:
   Rita will go to the store.
9. ANS:
   Cinderella is the one.
10. ANS:
    Let 2*x* and 2*y* represent any two even numbers. Their difference is $2x - 2y,$ or $2\left(x - y\right).$ Since 2 is a factor of this difference, $2x - 2y$ is even.
11. ANS:
    $(2n + 1)^2 = 4n^2 + 4n + 1 = 2\left(2n^2 + 2n\right) + 1$, which is an odd integer.
12. ANS:
    invalid; converse error (The figure could have been a triangle.)
13. ANS:
    Step 5: Divided by zero since a = b and then a - b = 0
14. ANS:

    a. Answer will vary

    b. The result will be the original number.

    c. Let n = the number.
    Then double the number is $2n$.

    Adding 6 yields $2n + 6$,
    dividing this sum by 2 gives $\frac{2n + 6}{2} = \frac{2(n + 3)}{2}$, or $n + 3$.
    Finally, subtracting 3 yields $(n + 3) - 3 = n$.

    Therefore, for any number *n,* the result is the original number.