

Mathematics 2201
Common Mathematics Assessment

June 12, 2013

Name: _____
Mathematics _____
Teacher: _____

Answer Key

28 Selected Response
13 Constructed Response

28 marks
42 marks

FINAL
TIME: 2 HOURS

70 Marks

NOTE

Diagrams are not necessarily drawn to scale.

FORMULAE

$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$	$a^2 = b^2 + c^2 - 2bc \cos A$	$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$
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$\sigma = \sqrt{\frac{\sum(x - \bar{x})^2}{n}}$	$z = \frac{x - \mu}{\sigma}$	$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
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Selected Response: Choose the appropriate response on the answer sheet or SCANTRON.

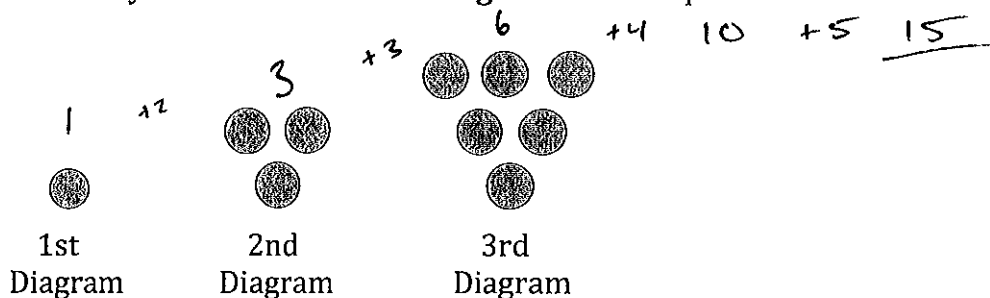
1. What is a statement that is believed to be true but not yet proven?

- (A) Conjecture
- (B) Counterexample
- (C) Deductive Reasoning
- (D) Inductive Reasoning

2. Which is a counterexample to the statement "The sum of two consecutive integers is always greater than each of the two integers"?

- (A) $-4 + (-5) = -9$
- (B) $4 + (-5) = -1$
- (C) $-4 + 5 = 1$
- (D) $4 + 5 = 9$

3. How many circles are in the 5th diagram in the sequence below:

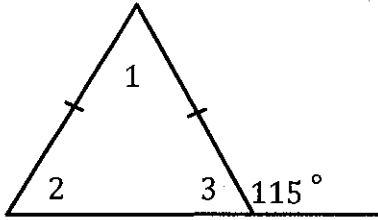


- (A) 9
- (B) 10
- (C) 14
- (D) 15

4. If two non-parallel lines are cut by a transversal, which pair of angles is always equal?

- (A) Alternate Interior
- (B) Corresponding
- (C) Supplementary
- (D) Vertically Opposite

5. A student was asked to find the measure of $\angle 1$. In which step did he make the first error?



Solution

Step 1: $\angle 3 = 180^\circ - 115^\circ$ ✓

Step 2: $\angle 3 = 65^\circ$ ✓

Step 3: $\angle 1 = \angle 3$ ✗

Step 4: $\angle 1 = 65^\circ$

- (A) 1
 (B) 2
 (C) 3
 (D) 4

6. How many sides does a convex polygon have if the sum of its interior angles is 1440° ?

- (A) 4
 (B) 6
 (C) 8
 (D) 10

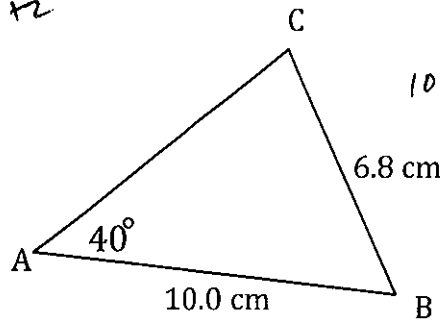
$$\frac{1440}{180} = \frac{180}{180}(n-2)$$

$$2 + 8 = n - 2 + 2$$

$$10 = n$$

7. What is the measure of $\angle C$?

- (A) 20°
 (B) 26°
 (C) 69°
 (D) 71°



$$10 \cdot \frac{\sin 40}{6.8} = \frac{\sin C}{10}$$

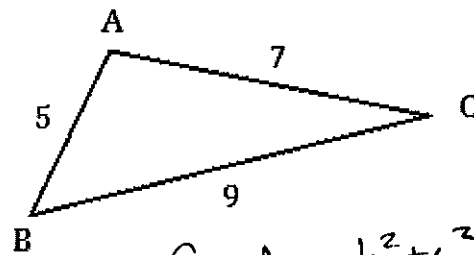
$$0.9453 = \sin C$$

$$C = \sin^{-1}(0.9453)$$

$$C = 71^\circ$$

8. Which equals the measure of $\angle A$?

- (A) $\cos^{-1}\left(\frac{5^2 + 9^2 - 7^2}{2(5)(9)}\right)$
 (B) $\cos^{-1}\left(\frac{7^2 + 5^2 - 9^2}{2(7)(5)}\right)$
 (C) $\cos^{-1}\left(\frac{9^2 + 5^2 - 7^2}{2(9)(5)}\right)$
 (D) $\cos^{-1}\left(\frac{9^2 + 7^2 - 5^2}{2(9)(7)}\right)$



$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

$$= \frac{7^2 + 5^2 - 9^2}{2(7)(5)}$$

9. Simplify completely:

$$5\sqrt{7} + 3\sqrt{28}$$

- (A) $11\sqrt{7}$
- (B) $17\sqrt{7}$
- (C) $11\sqrt{14}$
- (D) $8\sqrt{35}$

$$\begin{aligned} & 3\sqrt{4}\sqrt{7} \\ & 3(2)\sqrt{7} \\ & \rightarrow 5\sqrt{7} + 6\sqrt{7} \\ & 11\sqrt{7} \end{aligned}$$

10. Simplify completely:

$$\sqrt[3]{-8x^{17}}$$

$$17 \div 3 = 5 \text{ 2 remainder}$$

- (A) $-2x^2 \sqrt[3]{x^5}$
- (B) $-2x^5 \sqrt[3]{x^2}$
- (C) $2x \sqrt[3]{-2x^8}$
- (D) $2x^8 \sqrt[3]{-2x}$

$$x^5 \sqrt[3]{-8x^2}$$

$$\sqrt[3]{-8} = -2$$

$$-2x^5 \sqrt[3]{x^2}$$

11. Write $3x^3\sqrt{5x}$ as an entire radical.

- (A) $\sqrt{15x^4}$
- (B) $\sqrt{15x^7}$
- (C) $\sqrt{45x^4}$
- (D) $\sqrt{45x^7}$

$$\begin{aligned} 3^2 &= 9 \\ (x^3)^2 &= x^6 \end{aligned}$$

$$\begin{aligned} & 9x^6 \cdot 5x \\ & \sqrt{45x^7} \end{aligned}$$

12. A student was asked to simplify $\frac{x\sqrt{18x^3}}{3}$ but did not complete a correct solution. Which step contains her first error?

Solution:

Step 1:

$$\frac{x\sqrt{9 \cdot 2 \cdot x^2 \cdot x}}{3}$$

Step 2:

$$\frac{x \cdot 9x^2 \sqrt{2x}}{3}$$

Step 3:

$$\frac{9x^3 \sqrt{2x}}{3}$$

Step 4:

$$3x^3 \sqrt{2x}$$

- (A) 1
- (B) 2
- (C) 3
- (D) 4

13. Simplify completely:

$$\frac{5\sqrt{32}}{2\sqrt{3}}$$

(A) $\frac{10\sqrt{6}}{3}$

(B) $\frac{40\sqrt{6}}{3}$

(C) $\frac{5\sqrt{96}}{6}$

(D) $\frac{10\sqrt{96}}{12}$

$$= \frac{5\sqrt{16}\sqrt{2}}{2\sqrt{3}}$$

$$= \frac{5 \cdot 4\sqrt{2}}{2\sqrt{3}}$$

$$= \frac{10\sqrt{2}}{\sqrt{3}}$$

$$= \frac{10\sqrt{2}\sqrt{3}}{\sqrt{3}\sqrt{3}}$$

$$= \frac{10\sqrt{6}}{3}$$

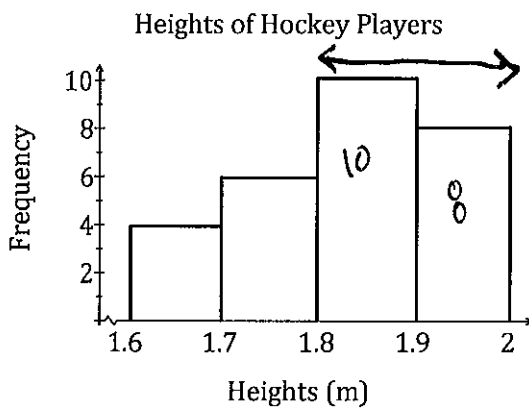
14. What are the restrictions on the variable for $\sqrt{x+2}$?

- (A) $x \geq -2$
 (B) $x > -2$
 (C) $x \geq 2$

$$x+2 \geq 0$$

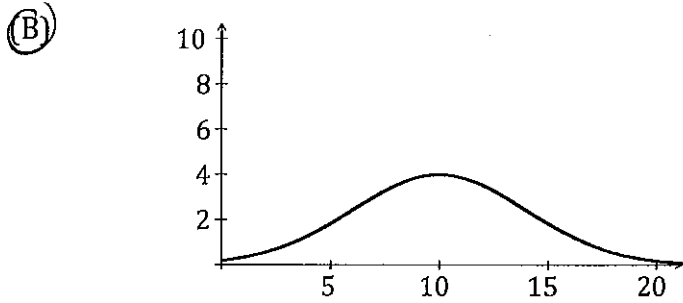
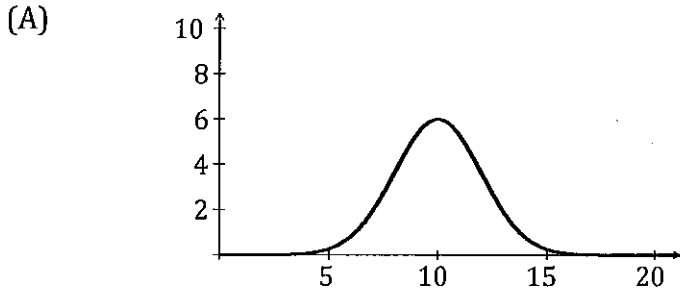
$$x \geq -2$$

15. The histogram shown represents the heights of hockey players on a professional hockey team. How many players have a height between 1.8 m and 2.0 m?

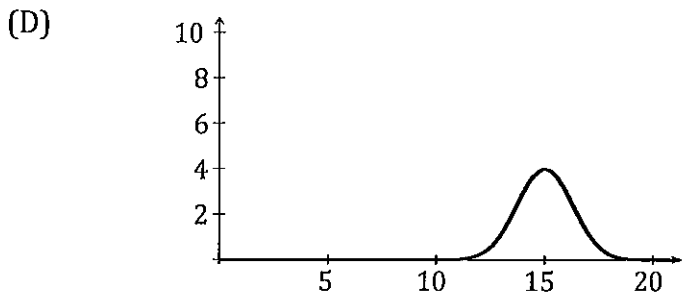
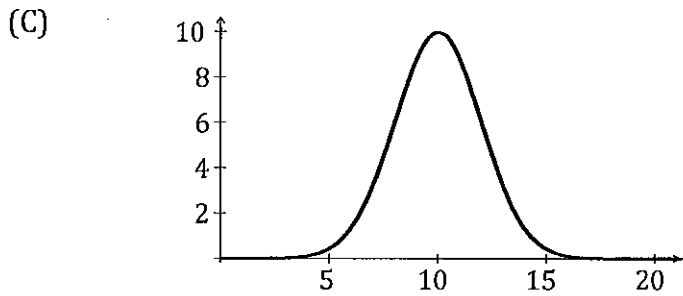


- (A) 10
 (B) 18
 (C) 24
 (D) 28
 (D) $x > 2$

16. Which represents data with the largest standard deviation?

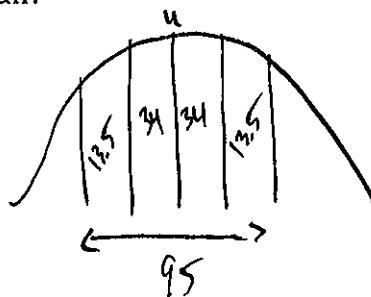


widest graph = largest sd.



17. A set of data is normally distributed. What percent of the data is within two standard deviations of the mean?

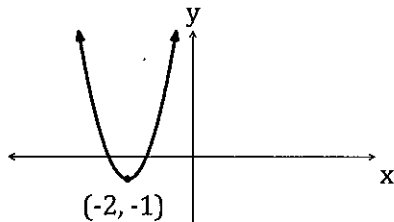
- (A) 47.5
- (B) 68
- (C) 95
- (D) 99.7



18. The function $y = -3x^2 - 12x - 13$ has axis of symmetry $x = -2$. Which represents the function?

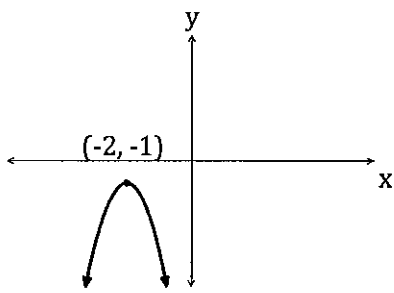
open down

(A)

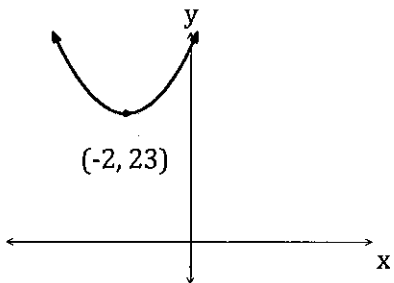


$$\begin{aligned}
 y &= -3(-2)^2 - 12(-2) - 13 \\
 &= -3(4) + 24 - 13 \\
 &= -12 + 24 - 13 \\
 &= -1 \\
 &\text{vertex } (-2, -1)
 \end{aligned}$$

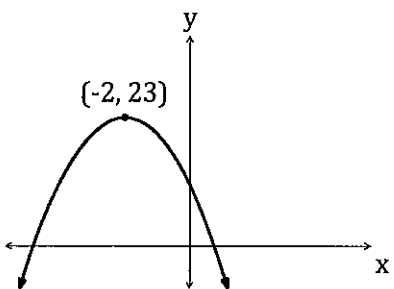
(B)



(C)



(D)



19. What is the domain and range for $f(x) = -2(x+1)^2 - 3$?

- (A) $x \in \mathbb{R}$ and $f(x) \leq -3$
 (B) $x \in \mathbb{R}$ and $f(x) \geq -3$
 (C) $x \leq -1$ and $f(x) \in \mathbb{R}$
 (D) $x \geq -1$ and $f(x) \in \mathbb{R}$

$x \in \mathbb{R} \rightarrow$ always Domain

vertex $(-1, -3)$

opens down \rightarrow less than vertex

$$f(x) \leq -3$$

20. A parabola has x-intercepts of $(-2,0)$ and $(-8,0)$. What is the axis of symmetry?

- (A) $x = -5$
- (B) $x = -3$
- (C) $y = -5$
- (D) $y = -3$

$$\frac{-8 + -2}{2} = \frac{-10}{2} = -5$$

21. What is the vertex of $y = 2x^2 + 8x - 5$?

- (A) $(-2, -29)$
- (B) $(-2, -13)$
- (C) $(2, 15)$
- (D) $(2, 19)$

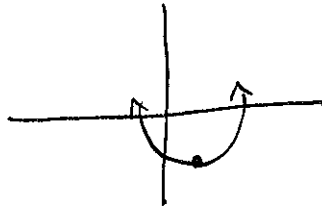
$$x = \frac{-b}{2a} = \frac{-8}{2(2)} = \frac{-8}{4} = -2$$

$$y = 2(-2)^2 + 8(-2) - 5 = 8 - 16 - 5 = -13$$

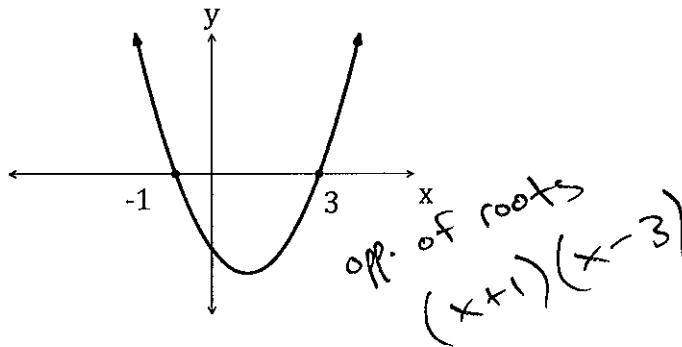
$(-2, -13)$

22. The graph of a quadratic function has vertex $(1, -4)$ and opens upward. How many x-intercepts does it have?

- (A) 0
- (B) 1
- (C) 2
- (D) 3



23. What is the equation of the function graphed below?



- (A) $y = (x - 1)(x - 3)$
- (B) $y = (x - 1)(x + 3)$
- (C) $y = (x + 1)(x - 3)$
- (D) $y = (x + 1)(x + 3)$

24. Which is a root of $2x^2 - 5x - 3 = 0$

- (A) -3
- (B) -1
- (C) 1
- (D) 3

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

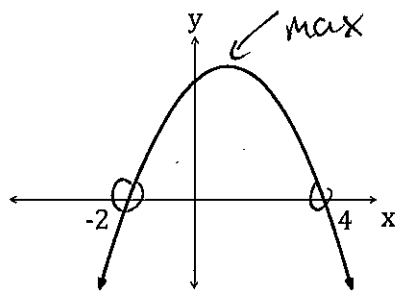
$$= \frac{5 \pm \sqrt{(-5)^2 - 4(2)(-3)}}{2(2)}$$

$$= \frac{5 \pm \sqrt{49}}{4} = \frac{5 \pm 7}{4}$$

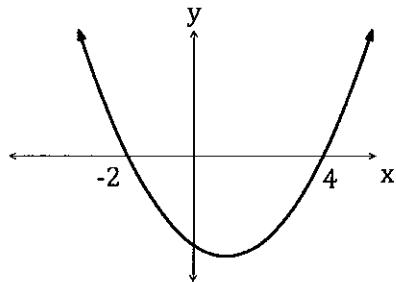
$x = 3$
 $x = -\frac{1}{2}$

25. Which represents a quadratic function with zeros of -2 and 4 and a maximum value?

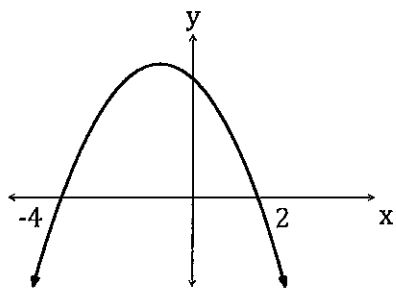
(A)



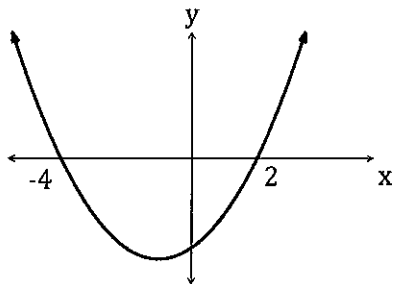
(B)



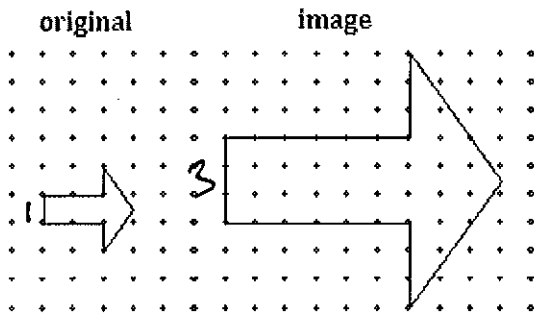
(C)



(D)

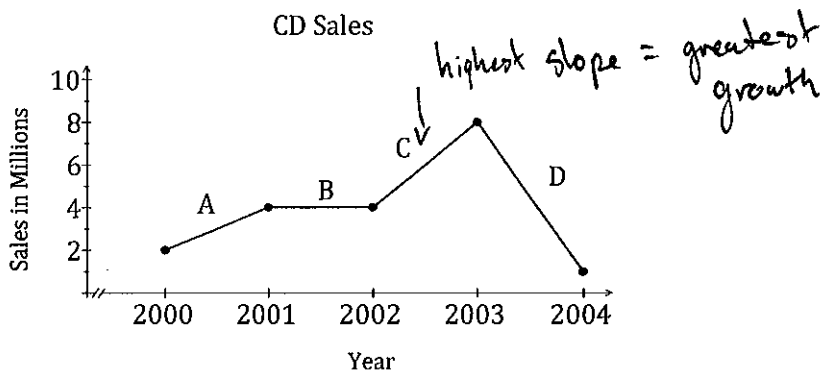


26. What is the scale factor in the figure below?



- (A) $\frac{1}{3}$
- (B) $\frac{1}{2}$
- (C) 2
- (D) 3
- $K = \frac{\text{scale}}{\text{orig}} = \frac{3}{1} = 3$

27. During which time period was the growth rate of CD sales the greatest in the graph shown?



- (A) 2000 - 2001
- (B) 2001 - 2002
- (C) 2002 - 2003
- (D) 2003 - 2004

28. The surface area of a cone is 34 ft^2 . If the cone is enlarged by a scale factor of 3, what is the surface area, in ft^2 , of the image?

- (A) 37
 (B) 102
 (C) 306
 (D) 918
- A. $(k)^2 \quad 34 \times 3^2$

Constructed Response:

Answers to be written on this paper in the space provided. Show all workings.

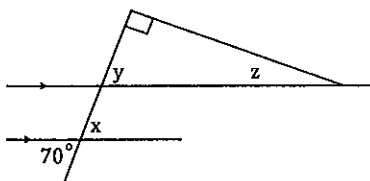
29. Use **both** inductive and deductive reasoning to show that the result for the given number trick will always be the original number. [4 marks]

NUMBER TRICK	Inductive Reasoning	Deductive Reasoning
Choose a number.	3	x
Double it.	6	2x
Add 6.	12	2x+6
Double it	24	4x+12
Subtract 4.	20	4x+8
Divide by 4.	5	x+2
Subtract 2.	3	x

The result will be the same as the number you started with.

30. Find the measure of each indicated angle. Justify your answer.

[3 marks]



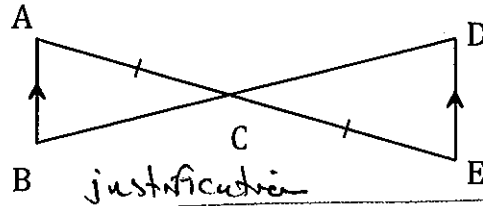
Angle Measure	Justification
$x = 70^\circ$	vertically opposite angles
$y = 70^\circ$	corresponding angles on parallel lines
$z = 20$	$180 - 70 - 90 = 20$ sum in $\Delta = 180^\circ$

31. Use either a paragraph or two-column format to complete the given proof:

[3marks]

Given: $AB \parallel DE$
 $AC = EC$

Prove: $\triangle ABC \cong \triangle EDC$



statement

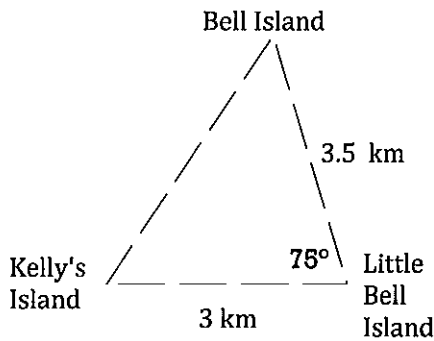
1. $AB \parallel DE$
2. $\angle BAC = \angle CED$
3. $\overline{AC} = \overline{EC}$
4. $\angle ACB = \angle DCE$
5. $\triangle ABC \cong \triangle EDC$

justification

Given.
 alternate interior angles
 given.
 vertically opposite angles
 by angle side angle

32. A boat travels from Bell Island to Kelly's Island to Little Bell Island, and returns directly back to Bell Island. What is the total distance travelled?

[4 marks]



$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$a^2 = 3^2 + 3.5^2 - 2(3)(3.5) \cos 75$$

$$a^2 = 9 + 12.25 - 5.4352$$

$$\sqrt{a^2} = \sqrt{15.8148}$$

$$a = 3.98 \text{ km} = 4.0 \text{ km}$$

$$\text{Total dist} = 3 + 3.5 + 4.0$$

$$= 10.5 \text{ km}$$

33. Simplify completely: $5\sqrt{6}(\sqrt{3} + 3\sqrt{12} - \sqrt{2})$ [3 marks]

$$\begin{aligned}
 &= (5\sqrt{6})(\sqrt{3}) + (5\sqrt{6})(3\sqrt{12}) - (5\sqrt{6})(\sqrt{2}) \\
 &= 5\sqrt{18} + 15\sqrt{72} - 5\sqrt{12} \\
 &= 5\sqrt{9}\sqrt{2} + 15\sqrt{36}\sqrt{2} - 5\sqrt{4}\sqrt{3} \\
 &= 5 \cdot 3\sqrt{2} + 15 \cdot 6\sqrt{2} - 5 \cdot 2\sqrt{3} \\
 &= 15\sqrt{2} + 90\sqrt{2} - 10\sqrt{3} \\
 &= 105\sqrt{2} - 10\sqrt{3}
 \end{aligned}$$

34. State the **restrictions** on x , **solve** the equation, and then **check** for extraneous roots. [4 marks]

$$\begin{aligned}
 \sqrt{3x+1} - 3 &= -4 & \text{rest } 3x+1 &\geq 0 \\
 & & x &\geq -1/3 \\
 \sqrt{3x+1} - 3 &= -4+3 \\
 (\sqrt{3x+1})^2 &= (-1)^2 & \text{check: } \sqrt{3(0)+1} - 3 \\
 3x+1 &= 1 & &= \sqrt{1} - 3 \\
 3x &= 0 & &= 1 - 3 \\
 x &= 0 & &= -2 \neq -4 \quad \text{extraneous} \\
 & & & \text{no. solution}
 \end{aligned}$$

35. A factory produces automotive brake pads with a mean mass of 174 g and a standard deviation of 0.7 g. Quality control expects that the mass of the pads will lie within the acceptable range of 173.9 g and 174.1 g. What is the confidence interval and margin of error this factory uses for its quality control tests? [2 marks]

$$\begin{aligned}
 \text{Con. int: } & 173.9 - 174.1 \\
 173.9 - 174 &= -0.1 \\
 174.1 - 174 &= 0.1 \\
 \text{Margin of error is } & \pm 0.1
 \end{aligned}$$

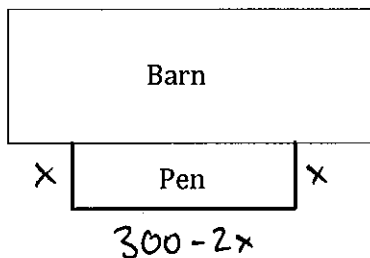
36. Jason scored 82% on a test where the class average was 74% and the standard deviation was 10.6%. If the class was normally distributed, what percentage of the class scored better than Jason? [3 marks]

$$z \text{ score} = \frac{x - \mu}{\sigma} = \frac{82 - 74}{10.6} = 0.75$$

Table 0.75 gives 0.7734, gives 77.34% below

$$100 - 77.34 = 22.66\% \text{ scored above Jason.}$$

37. A farmer has 300 m of chain link fencing to create a rectangular pen, using the side of a barn as one side of the pen. Algebraically determine the maximum area that can be enclosed by the pen. [4 marks]



$$\begin{aligned} A &= l \times w \\ &= x(300 - 2x) \\ &= 300x - 2x^2 \end{aligned}$$

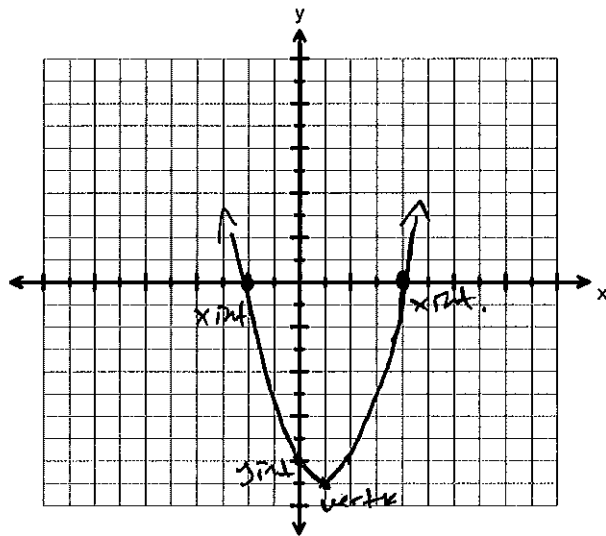
$$\begin{aligned} \text{max at vertex } x &= \frac{-b}{2a} = \frac{-300}{2(-2)} \\ &= \frac{-300}{-4} \\ &= 75 \end{aligned}$$

max width is 75 m

$$\begin{aligned} \text{max area: } w &= 75 \\ l &= 300 - (2)(75) \\ &= 150 \end{aligned}$$

$$\begin{aligned} A &= l \times w = 75 \times 150 \\ &= 11250 \text{ m}^2 \end{aligned}$$

38. Algebraically determine the **vertex** and **x-intercepts** for the function $y = x^2 - 2x - 8$. Sketch the graph, labelling all key points. [3 marks]



$$\text{vertex: } x = \frac{-b}{2a} = \frac{-(-2)}{2(1)} = \frac{2}{2} = 1$$

$$y = (1)^2 - 2(1) - 8 = 1 - 2 - 8 = -9 \quad \text{vertex } (1, -9)$$

$$y \text{ int: } (0, -8)$$

$$x \text{ int } y = (x-4)(x+2) \\ x = 4 \quad x = -2$$

39. Solve the given equation. State the solution(s) in **exact** form. [3 marks]

$$6x^2 = -4x + 3$$

$$6x^2 + 4x - 3 = 0$$

$$a = 6$$

$$b = 4$$

$$c = -3$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-4 \pm \sqrt{(4)^2 - 4(6)(-3)}}{2(6)}$$

$$x = \frac{-4 \pm \sqrt{88}}{12}$$

$$x = \frac{-4 \pm \sqrt{4} \sqrt{22}}{12}$$

$$x = \frac{-4 \div 2 \pm 2 \sqrt{22}}{12 \div 2}$$

$$x = \frac{-2 \pm \sqrt{22}}{6}$$

40. On another planet, the path of a rock that is thrown is given by $h = -t^2 + 4t + 6$, where h is height in metres and t is time in seconds. At what time(s) would the height of the rock be 9 m? [3 marks]

$$9 = -t^2 + 4t + 6$$

$$0 = -t^2 + 4t - 3 \quad a = -1$$

$$b = 4$$

$$c = -3$$

$$* = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$t = \frac{-4 \pm \sqrt{(4)^2 - 4(-1)(-3)}}{2(-1)}$$

$$t = \frac{-4 \pm \sqrt{4}}{-2}$$

$$t = +3$$

$$t = +1$$

The rock would be @ 9m high @ 1s and 3s.

41. Avalon Supermarket sells a box of 48 granola bars for \$7.99 and a box of 8 bars for \$1.99. What is the least expensive way to buy 70 granola bars? Justify your reasoning. [3 marks]

at least 70 bars : 3 cases : ① 2 x 48 box : 2 x 7.99 = \$15.98
96 bars

② 9 x 8 box : 9 x 1.99 = \$17.91
72 bars

③ 1 x 48 box ; 3 x 8 box : 1 x 7.99 + 3 x 1.99
72 bars : = 7.99 + 5.97
= \$13.96

option 3 is best option