Unit VI

Exponential Functions

Suggested Time

12 hours = 12%

In Mathematics 1201, students worked with linear relations expressed in slope-intercept form, general form and slope-point form (RF6). In Mathematics 2201, they solved problems that involved quadratic equations (RF2). Students will now be exposed to exponential equations of the form $y = a(b)^x$, where $b > 1$ or $0 < b < 1$ and $a > 0$. Work with logarithmic functions will be completed in the next unit.

Exponential function: any function of the form

$y = a \bullet (b)^x$ Where a>0, b > 0, b≠1

Case 1: b > 1 (Exponential Increasing Curve or Exponential Growth Curve)

As x increases y increases without bound

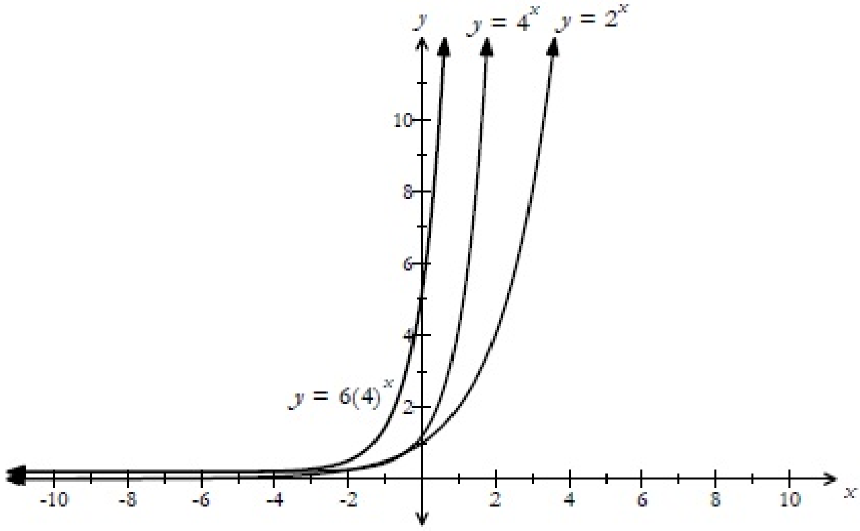
ex) Sketch the curves

$i)\ y = 6(4)^x \quad ii)\ y = 4^x \quad iii)\ y = 2^x$

for a table of values for x = -3 to 3

x	$y = 6(4)^x$	$y = 3^x$	$y = 2^x$
-3			
-2			
-1			
0			
1			
2			
3			
4		81	16

6-1 to 6-2 Feb



End Behaviour:
all graphs increase without bound as x gets larger (increasing curves----rises to the right as x increases) these are known as exponential GROWTH curves b>1)

asymptote: the horizontal line an exponential curve does not touch. Here it is the x-axis for all three cases. The equation of the asymptote is y = 0. This means the graph will never hit y = 0. (Why later?)

These curves are all growth curves and the larger the value of B the faster the growth rate. These curves are all exponential growth curves...increasing curves.

Domain: $x \in R$ Range: $y > 0$

Range: Affected by the Horizontal asymptote [y = 0]
Exponential Curves Continued

Case II $0 < b < 1$ (Decay curves : as x increases y is decreasing forever) $y = a(b)^x$

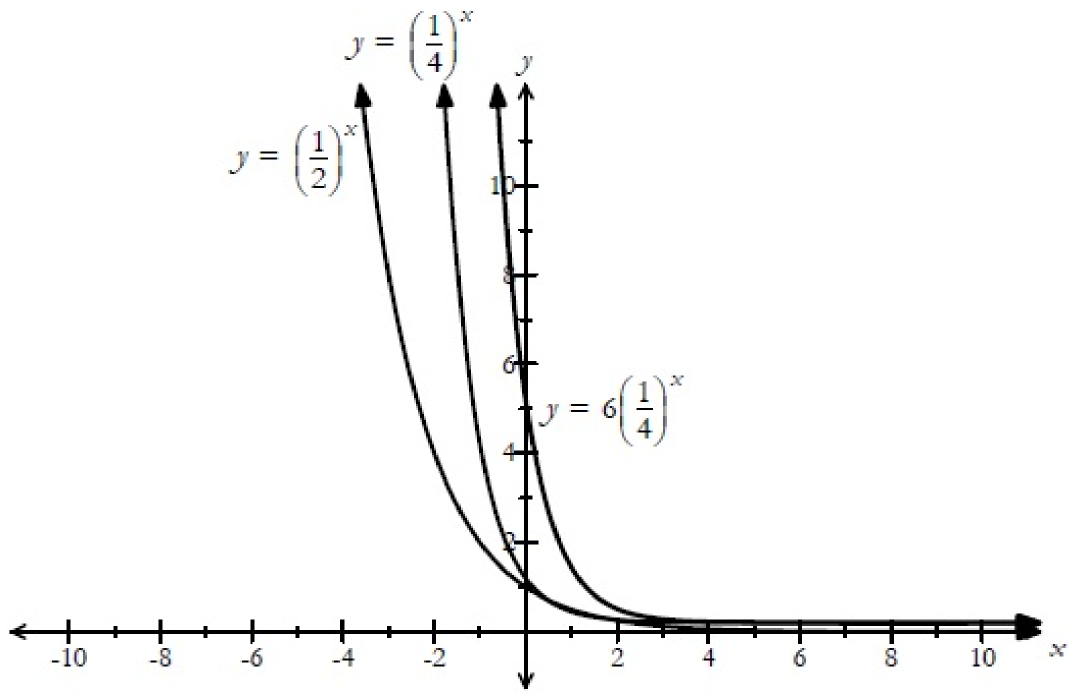
ex) Sketch the exponential functions for

$y = \left(\frac{1}{2}\right)^x$, $y = \left(\frac{1}{4}\right)^x$, $y = 6\left(\frac{1}{4}\right)^x$

x	$y = (\frac{1}{2})^x$	$y = \left(\frac{1}{4}\right)^x$	$y = 6\left(\frac{1}{4}\right)^x$
-3		64	384
-2			
-1			
0			
1			
2			
3			
4			

Domain: Range:

Asymptote: y = ______



End Behaviour: decreasing curves as x increases y is decreasing

Asymptote: y = 0 (Horizontal Line the graphs do not touch or cross)

Domain: $\{x / x \in R\}$ Range $\{y / y > 0, y \in R\}$

Fill in the information for the exponential curves below:

	$y = 2^x$	$y = 3^x$	$y = 5(3)^x$
y-intercept			
number of x-intercepts			
end behaviour			
domain			
range			

	$y = (\frac{1}{2})^x$	$y = (\frac{1}{4})^x$	$y = 3(\frac{1}{4})^x$
y-intercept			
number of x-intercepts			
end behaviour			
domain			
range			

Summary: $y = a \bullet (b)^x$

1) if b > 1 exponential growth (increasing curve)
2) (0,a) = y-intercept
3) if 0 < b < 1, exponential decay curve (decreasing curve)
4) HA: y = 0

NOTE: $y = (-2)^x$

5) if b < 0 graph oscillates back and forth between points on y

6) if b = 1, $y = a \bullet (b)^x$ its just the horizontal line y = a which is a constant function

7) So exponential functions can never have the base (b) in $y = a \bullet (b)^x$ being (1) 0 or negative (2) b = 1

page 337 1, 2, 3

Characteristics of an Exponential Equation   6-2

Model: $y = a(b)^x$

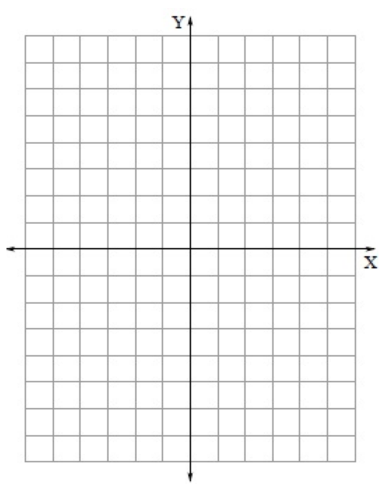
- What is the relationship between the *y*-intercept and the parameter *a*?
- How can the *y*-intercept be determined algebraically using the equation?
- What happens to the exponential function if $b > 1$? What happens to the *y*-values as you move from left to right on the *x*-axis?
- What happens to the exponential function if $0 < b < 1$? What happens to the *y*-values as you move from left to right on the *x*-axis?
- Why does the graph have no *x*-intercepts?
- Is the domain or range affected by changing the parameters *a* or *b*?

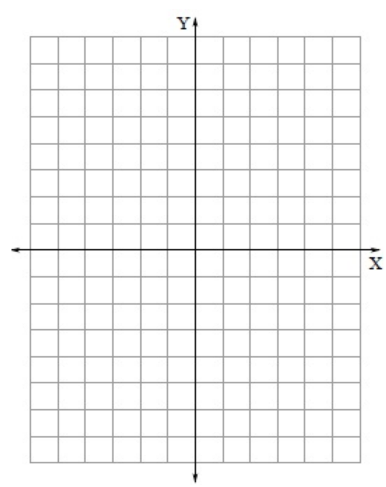
This would be a good opportunity to investigate the graph of $y = b^x$ where $b = 1$ and where $b < 0$. They should observe the following:

- When $b = 1$, a horizontal line is produced.
- When *b* is negative, if integer values of *x* are chosen the *y*-values "oscillate between positive and negative values; for rational values of *x*, non-real values of *y* may be obtained.

Ex) How is $y = x^2$ different from $y = 2^x$?

Rough Sketches





Domain:

Range:

End Beh:

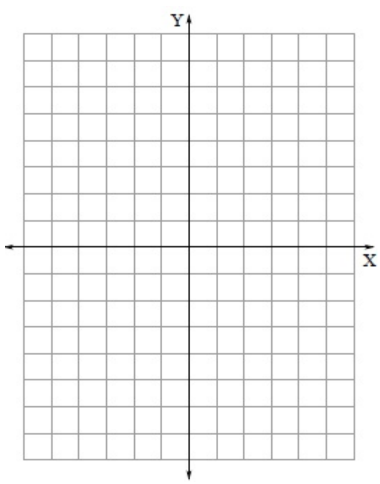
x-inter:

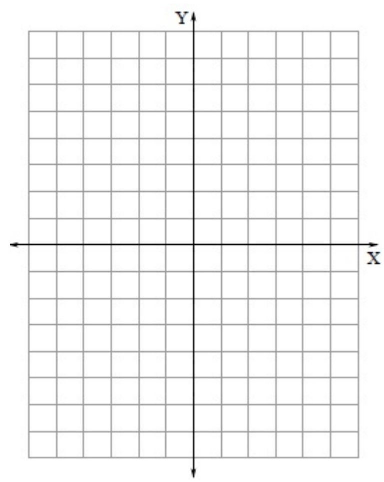
Asymptote:

- Ask students to complete the following table using $y = 8\left(\frac{2}{3}\right)^x$:

$y = 8\left(\frac{2}{3}\right)^x$	True	False	Why I think so
(i) the y-intercept is 1			
(ii) the graph has one x-intercept			
(iii) the range is $\{y \mid y > 0, y \in R\}$			
(iv) the domain is $\{x \mid x > 8, x \in R\}$			
(v) this is a decreasing exponential function			

- Using a table of values, ask students to graph $y = (-2)^x$ and $y = (1)^x$. They should explain why the graphs do not represent exponential functions.





Reason: base = b =   For y = (1)

Summary: $y = a \bullet (b)^x$

1) if b > 1 exponential growth (increasing curve)
2) (0,a) = y-intercept
3) if 0 < b < 1, exponential decay curve (decreasing curve)
4) HA: y = 0
5) if b < 0 graph oscillates back and forth between points
6) if b = 1, $y = a \bullet (b)^x$ its just the horizontal line y = a

Page 337 1,2,3
Page 347 1 (Scatter plots in TI-83), 2,3,4,5, 6, 7, 8, 9

Solving Exponential Equations 6-3

Exponential equations: variable is in the exponent

The basic premise to get the same base on the each side of the equation and then drop or cancel the bases from each side of the equation and solve.

Case 1 Like Bases

ex) Solve for x: A) $2^{x+7} = 2^{2x+8}$; $2^{x-3} = 2^{-x}$

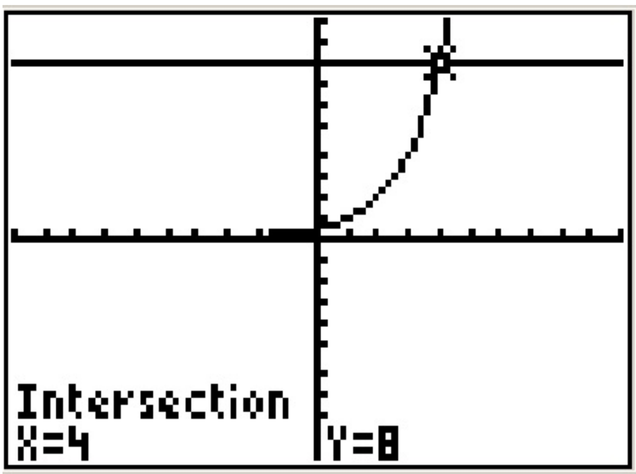
B) $\left(\frac{1}{3}\right)^{2x-1} = \left(\frac{1}{3}\right)^{4x-7}$

C) $7^{-2x+1} - 7^{x-5} = 0$

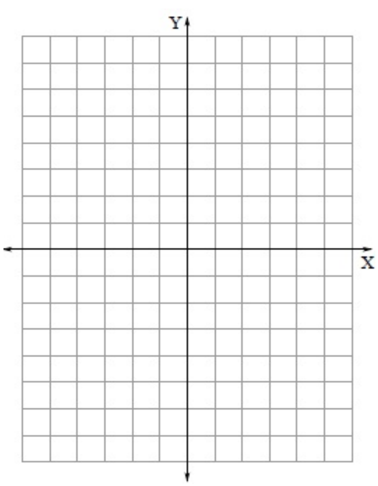
Case II Unlike Bases on each side:

Visual: What does $2^{x-1} = 8$ mean? Bases are different!

Graphically: where do the two graphs intersect?



So when we solve $2^{x-1} = 8$ we are trying to see where the exponential function on the left (Growth Exponential $y = 2^{x-1}$) is meeting or intersecting the Constant function on the right y = 8



Ex) Solve: $3^{x-2} = 9$ Rough Sketch below

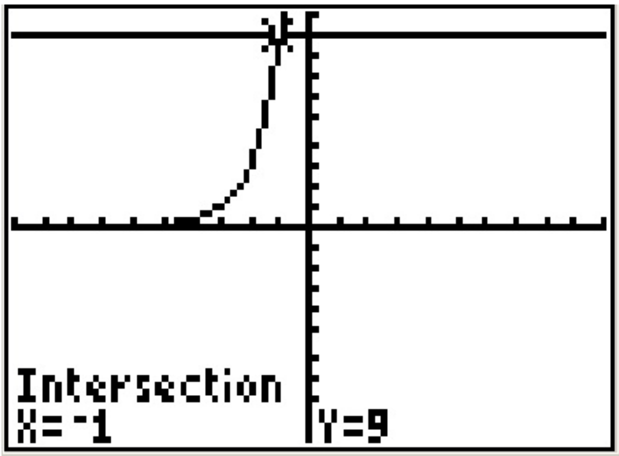
Now Note algebraically:
(Get the same base on each side)

$3^{x-2} = 9$

Exponential Equations with Different Bases CTED 6-3

Solve for x: graphically: $3^{x+3} = 9$

Note algebraically:



Solving algebraically: If the bases are different you must get a common base on EACH side DOP LA 3-4%

Common Exponent Laws You MUST know:

1) $a^0 = 1$

2) $a^1 = a$

3) *i)* $a^x \bullet a^y = a^{\square}$ *ii)* $a^x \div a^y = a^{\square}$ *iii)* $(a^x)^y = a^{\square}$

Ex i) Ex ii) Ex iii)

4) Negative Exponents:

i) $a^{-n} = \frac{\square}{\square}$ *ii)* $\frac{1}{a^{-n}} = \frac{\square}{\square} = \square$ *iii)* $\left(\frac{a}{b}\right)^{-n} = \left(-\right)^n$

Law	Example
$x^1 = x$	$6^1 = 6$
$x^0 = 1$	$7^0 = 1$
$x^{-1} = 1/x$	$4^{-1} = 1/4$
$x^m x^n = x^{m+n}$	$x^2 x^3 = x^{2+3} = x^5$
$x^m/x^n = x^{m-n}$	$x^6/x^2 = x^{6-2} = x^4$
$(x^m)^n = x^{mn}$	$(x^2)^3 = x^{2\times 3} = x^6$
$(xy)^n = x^n y^n$	$(xy)^3 = x^3 y^3$
$(x/y)^n = x^n/y^n$	$(x/y)^2 = x^2 / y^2$
$x^{-n} = 1/x^n$	$x^{-3} = 1/x^3$
And the law about Fractional Exponents:	
$x^{\frac{m}{n}} = \sqrt[n]{x^m} = (\sqrt[n]{x})^m$	$x^{\frac{2}{3}} = \sqrt[3]{x^2} = (\sqrt[3]{x})^2$

$$\left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^{n} \quad so \quad \left(\frac{2}{3}\right)^{-3} = \left(\frac{3}{2}\right)^{3} = \frac{3^3}{2^3} = \frac{27}{8}$$

Solving algebraically: If the bases are different you must get a common base on EACH side DOP LA 3-4%

$A)\ 4^{3-x} = 2^{3x+9}$ $B)\ 3^{4x+9} = 27^{x-2}$ $C)\ 5^{x-6} = \frac{1}{125}$

$D)\ 2(4)^{2x} = \frac{1}{32}$

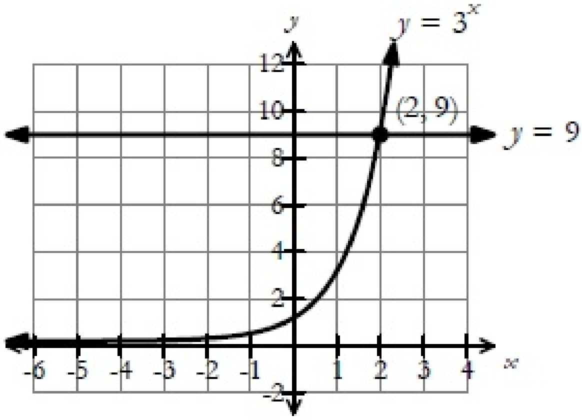
E) $5^{x+2} = \sqrt{125}$

F) $64^{x-1} = 32^{4-x}$

G) $8^{4x+2} = \sqrt[3]{2}$

What does $3^x = 9$ mean graphically?

Where the increasing exponential curve hits the constant function (line)



Ex) Solve for x: $3^{X+1} = 9$ graphically

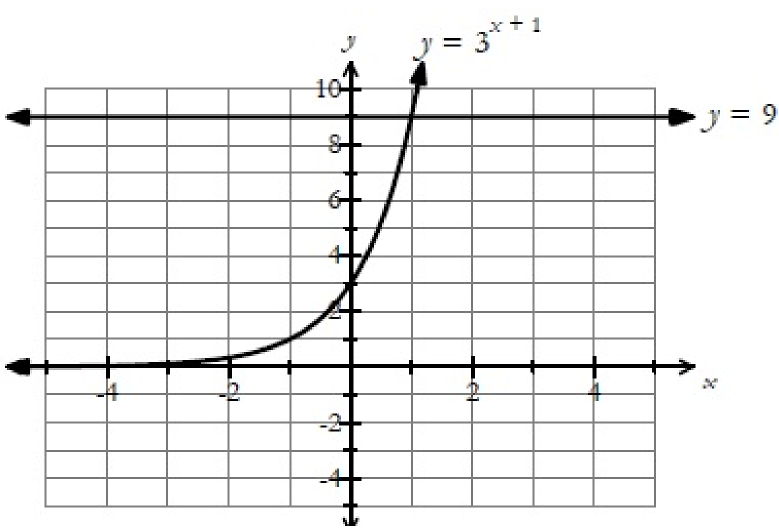


table of values

verify algebraically:

$$3^{X+1} = 9$$

Simplify the following: $5^{2x-2} = \left(\frac{1}{25}\right)^{x-1}$.

(A) $5^{2x-2} = 5^{3x-3}$

(B) $5^{2x-2} = 5^{-3x+1}$

(C) $5^{2x-2} = 5^{-3x-3}$

(D) $5^{2x-2} = 5^{-3x-+3}$

Applications of Exponential functions

Exponential expression arises: when a quantity changes by the same factor each time (the base)

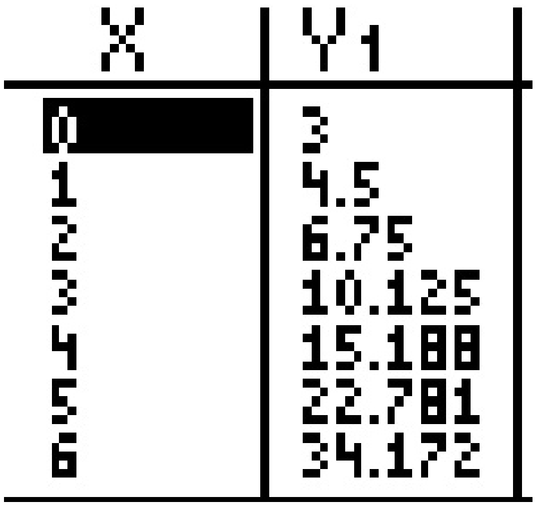
IE

x 1	2	3	4	5
y 4	12	36	108	324

common factor = base= growth or decay:

eq:

x 0	1	2	3	4
y 125	25	5	1	.2



X	Y1
0	3
1	4.5
2	6.75
3	10.125
4	15.188
5	22.781
6	34.172

common factor = base= growth or decay:

eq:

x -2	0	2	4	6
y 3	9	27	81	243

common factor = base= growth or decay:

6-4 Applications of Equations of Exponential Functions CTED
Date:

Model: $y = a(b)^x$ when increments in x are 1

1) Growth: b > 1 [Increasing Exponential]

x	0	1	2	3
y	5	10	20	40

For $y = a(b)^x$, a = 5 since y-intercept = a = 5 (when x = 0)
b = base = 10/5=20/10=40/20= 2 [latter/previous]

Therefore Model that describes this data is: $y = a(b)^x = 5(2)^x$

2) Decay: 0 < b <1 [Decreasing Exponential]

x	0	1	2	3
y	200	100	50	25

For $y = a(b)^x$, a = 200 since y-intercept = a = 200 (when x=0)
b = base = 100/200 = 50/100 = 25/50 = .5 [latter/previous]
Therefore Model that describes this data is: $y = a(b)^x = 200(.5)^x$

When the increments of x ARE NOT 1. Very similar one small change.

Model: y = $a(b)^{\frac{x}{c}}$

Where a = y-intercept b = base c = INCREMENTS in x (what x is going up by?)

Notice the new Model when increments of x are not 1 : y = $a(b)^{\frac{x}{c}}$

x -6	-3	0	3	6
y 100	50	25	12.5	6.25

NoTe: a = 25 (y-int when x = 0) , base = 50/100 = 25/50 = .5, c = 3 (increments in x now are 3 NOT 1)

Model (Equation): y = $a(b)^{\frac{x}{c}}$ = [Note where the 3 goes]

Check: if x = 6 answer should be y = 6.25

$$y = 25(.5)^{\frac{x}{3}}, \text{ if } x = 6, y = 25(.5)^{\frac{6}{3}} = 25(.5)^2 = 6.25$$

Another Example of when the increments of x ARE NOT 1.

Ex) A trout population grows according the table below where x is time in years and y is the amount present.

x	0	4	8	12
y	1200	3000	7500	18,750

A) Determine an exponential equation to represent the trend in this data.

Model: $y = a(b)^{\frac{x}{c}}$

a = 1200 b = $\frac{3000}{1200} = \frac{7500}{3000} = \frac{18750}{7500}$=2.5 c = 4 (increments in x)

Therefore: $y = 1200(2.5)^{\frac{x}{4}}$

B) What is the population after 10 years?

Answer: sub in 10 for x:

$$y = 1200(2.5)^{\frac{x}{4}} = 1200(2.5)^{\frac{10}{4}} = 11,858.54 = 11859 trout$$

6-4 Applications of Exponential Functions

"The mathematics of uncontrolled growth are frightening. A single cell of the bacterium E. coli would, under ideal circumstances, divide every twenty minutes. That is not particularly disturbing until you think about it, but the fact is that bacteria multiply geometrically: one becomes two, two become four, four become eight, and so on. In this way it can be shown that in a single day, one cell of E. coli could produce a super-colony equal in size and weight to the entire planet Earth."

Michael Crichton (1969) *The Andromeda Strain,* Dell, N.Y. p247

When diseases grow exponentially (H1N1, Aids (when it first came on the scene in the 80s...but in Africa today is still growing exponentially...the World Health Organization will often intervene....IE...SARs virus 10 years ago in Toronto, Bird Flu, West Nile Virus, Zika Virus lately...etc),

Applications types we will deal with are:

Growth: Investments (money), bacterial growth, populations, Doubling time (later)

Decay: vehicle depreciation, half-life (carbon-dating—later), Applications of Exponential Growth Continued 6-4

App 1) Money Investments (Exponential Growth b > 1)

Base in the function must be larger than 1

Model: $A = P(1 + i)^n$

Parts: A = Amount present after time n
P = Principle invested
i = interest converted to (Converted to dec)

n= number of investing periods

Ex 1

Shelly initially invests $500 and the value of the investment increases by 4% annually.

- Create a function to model the situation.
- How much money is in Shelly's investment after 30 years?
- What amount of time will it take for the investment to double?

Model: $A = P(1+i)^n$ given on info sheet

Interest on savings or loans can be paid

1) yearly (once a year) Annually i = interest per year
2) Semi-annually (twice year) i/2
3) Quarterly (4 times a year) i/4
4) Monthly (12 times a year) i/12
5) Semi-Monthly (twice a month 1st and 15th = 24 times a year) i/24
6) bi-weekly (26 times a year) i/26
7) weekly (52 times a year) i/52
8) daily (365 times a year) i/365

For $A = P(1+i)^n$ n = the number of compounding periods over the time period of the investment while I is the interest

interest I in the formula is computed according to:

Ex 2 A Wayne Gretzky rookie card increases in value 2% every year. Recently, a card sold for $94,613 at auction. Determine the value of the card 12 years from now using an exponential model.

Ex) A bacterial colony grows at a rate of 20% every hour. If the initial amount of bacteria are 1200 cells, fill in the table below and use it to determine an exponential model (equation) to describe the pattern of growth. Use it to determine how many bacteria will be present 28 hours later.

time hrs (x)	0	1	2	3	4
Amt. Of Bacteria present (Y)					

Review to apps to date with Exponential Functions:

Find an exponential equation to represent the following sets of data 2 ways (Algebraically and Using an Exponential Regression)

A)

x	-1	0	1	2	3
y	2.7	4	6	9	13.5

B)

x	0	3	6	9	12
y	90	180	360	720	1440

Common ratio = base = _______ Initial = ______(y-int.)

App 2 Doubling Time Problems (Growth)

If a quantity is doubling every so often...like in the E. Coli bacteria opening statement...then we use a doubling time model for the exponential function

Model: given on info sheet

$$A(t) = A_o(2)^{\frac{t}{h}}$$

Explain and label parts

ex 1 Doubling time

> The population of trout growing in a lake can be modeled by the function $P(t) = 200(2)^{\frac{t}{5}}$ where $P(t)$ represents the number of trout and t represents the time in years after the intial count. How long will it take for there to be 6400 trout?

Fill in:

t	0	1	2			
P(t)						

Solution:

Common Bases Why?

ex 2

A bacterial culture doubles in size every 12 hours. If there are 300 bacteria initially present,

A) determine how many will be present 2 days from now.
B) Algebraically, how long will it take for the culture to reach 19,200?

Model:

Ex) The doubling time of a certain bacterial colony is 4 hours. If there are 600 cells initially present, A) write an exponential model to describe the situation B) How many bacteria will be present after 7.5 hours C) Algebraically determine how many days it will take to reach a population of 38, 400?

App 3 Exponential Decay Applications 6-5

Depreciation (Value of a vehicle, houses, investments (loss), radioactive decay = half-life)

Model $A=A_o(1 - r)^x$....not given...its not there

Ex) A car depreciates yearly by 20%. If the car was purchased for $24,500...determine a model to represent the value of the car at any time t. Use it to determine the value of the car after 4 years? When will the car be worth approximately $8000?

Ex) A skidoo depreciates at a rate of 30% per year. If its purchase price was $18,500 in 2016, determine how much the skidoo is worth in 2019.

Ex) A house is currently valued at $275,000. It is estimated that it will depreciate at a rate of 18% per year for the next 10 years due to market decline. Determine an exponential model for the decreasing value and use it to determine the value of the house after 8 years.

App 4 Half-life—>when an isotope or radioactive element decays it has a "half-life"—drugs as well! It is an application of Exponential Decay

Ex) The half-life of Tylenol (acetaminophen) is 2-3 hours.

IE this means a 650 mg dosage will take ____hrs to decay down to ____mg

Special type of Exponential Depreciation [The half-life]

Half-life: how long (time) it takes for an element (object) to decrease to halve of its initial value or amount

Barium-122 has a half-life of 2 minutes. A fresh sample weighing 80 g was obtained. If it takes 10 minutes to set up an experiment using barium-122, how much barium-122 will be left when the experiment begins?

Every half-life, 2 minutes, half of the original amount will undergo nuclear decay:

Time:	start	2 min	4 min	6 min	8 min	10 min
Mass:	80 g	40 g	20 g	10 g	5 g	**2.5 g**

At the end of 10 minutes (5 half-lives) only **2.5 g** are left, the rest has decayed.

Model:

$$A(t) = A_o\left(\frac{1}{2}\right)^{\frac{t}{h}}$$

Half-life is a mathematical and scientific description of exponential or gradual decay.

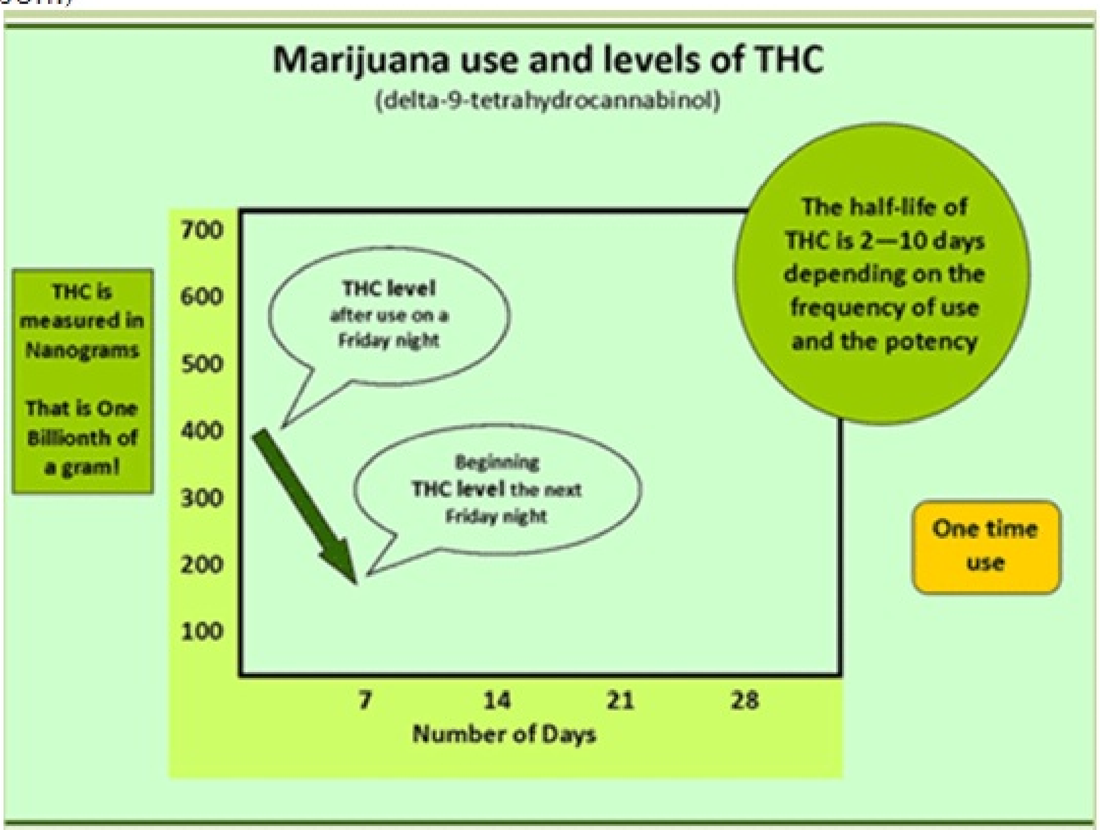
Look up ***half-life*** in Wiktionary, the free dictionary.

Half-life may also refer to:

- Biological half-life, the time it takes for a substance to lose half of its pharmacologic, physiologic, or radiologic activity or y =a $\left(\frac{1}{2}\right)^{\frac{x}{h}}$ label parts

Give real examples of half-life: aspirin: 3.1 to 3.2 hours, hard drugs (street)= cocaine: 40 minutes to 1 hour

(The THC levels used for this illustration are accurate as to actual expected levels in a person.)



Carbon-14 has a half-life of 5730 years and is used to date archaeological objects.

IE   dating dinosaur bones, plants, the ice man in the Alps

Over 5000 years old!!!!!

Half-Life Decreasing Exponential Apps Continued

Ex)

a. Iodine-131 is used to destroy thyroid tissue in the treatment of an overactive thyroid. The half-life of iodine-131 is 8 days. If a hospital receives a shipment of 200 g of iodine-131, how much I-131 would remain after 32 days?

Ex)

The half-life of a radioactive isotope is 30 hours. The amount of radioactive isotope $A(t)$, at time t, can be modelled by the function $A(t) = A_0\left(\frac{1}{2}\right)^{\frac{t}{h}}$. Ask students to determine algebraically how long it will take for a sample of 1792 mg to decay to 56 mg.

Ex) The half-life of a 650 mg dosage of aspirin is 3.2 hours. Using the model $A = A_0\left(\frac{1}{2}\right)^{\frac{t}{h}}$, determine how much will remain in your system after 8 hours. How long algebraically will it take to decay to 81.25 mg?

$$A = A_0 \left(\frac{1}{2}\right)^{\frac{t}{h}}$$

$$81.25 = 650\left(\frac{1}{2}\right)^{\frac{t}{3.2}}$$

$$\left(\frac{1}{8}\right) = \left(\frac{1}{2}\right)^{\frac{t}{3.2}}$$

$$\left(\frac{1}{2}\right)^{3} = \left(\frac{1}{2}\right)^{\frac{t}{3.2}}$$

$$3 = \frac{t}{3.2}$$

$$therefore\ t = 3x3.2 = 9.6\ hours$$

Application 6 Exponential Regression Models (TI-83—stat Calc___)

Fitting an exponential curve to a scatter plot and make predictions

Ex) The following table shows the household expenditures in NL for the period of 2000 to 2009.

Year	2000	2001	2002	2003	2004	2005	2006	2007	2008	2009
Household Expenditure	43 501	45 759	46 597	47 944	49 126	52 306	53 939	55 007	57 713	57 605

Determine a model to represent the data. If the trend continues, predict what will be the expenses in 2020.

Convert table so that it starts at time 0 from year 2000

t	0 (year 2000)	1	2	3	4	5	6	7	8	9L1
H(t)	43,501									L2

Set up scatter plot (PLOTS ON NOW!)

Model:=________________ Prediction for 2020

Algebraically : Calc:

Another example: Exponential regression

The following table shows the average undergraduate university tuition fees paid by full time Canadian students.

School Year	Tuition Fee
2004-2005	$4140
2005-2006	$4214
2006-2007	$4400
2007-2008	$4558
2008-2009	$4747
2009-2010	$4942
2010-2011	$5146
2011-2012	$5366

Ask students to respond to the following:

(i) Using graphing technology, construct a scatter plot to display the data.

(ii) Does your graph appear to have an exponential curve pattern?

(iii) Use an exponential regression to define a function that models the data.

(iv) Estimate the tuition fee for the school year 2015-2016.

Application 7

Simple verus Compound Interest (Exponential Growth)

Simple Interest: interest is paid yearly at a set rate but only on the original investment (NO BANK DOES THIS)

<u>A=P(1+rt) linear</u>

P = amount initially invested r = interest rate t = time in years

Consider investing $1000 for 6 years at 5% simple interest **(Re-visited in U9)**

Simple Interest:

A=1000(1+.05t)

Year (*t*)	Total Amount at the End of the Year (*A*)
0	$1000
1	$1050 ($1000 + $1000(0.05)(1))
2	$1100 ($1000 + $1000(0.05)(2))
3	$1150 ($1000 + $1000(0.05)(3))

note: relationship is LINEAR not Exponential. Why!

Kyle invested his summer earnings of $5000 at 8% simple interest, paid annually. Ask students to graph the growth of the investment for 6 years using "time (years)" as the domain and "value of the investment" as the range. They should answer the following:

(i) What does the shape of the graph tell you about the type of growth? Why is the data discrete?

(ii) What do the *y*-intercept and slope represent for the investment?

(iii) What is the value of the investment after 10 years?

Practice

note: discrete=don't join the points!...graph is a linear set of dots

Compound Interest

is determined by applying the interest rate to the sum of the principal and any accumulated interest

Consider: A $1000 investment invested for 6 years compounded annually at 5% interest

Year (t)	Amount of Annual Interest	Total Amount at the End of the Year (A)
0		$1000
1	$1000 \times 0.05 = 50$	$1050 $(1000(1.05)^1)$
2	$1050 \times 0.05 = 52.50$	$1102.50 $(1000(1.05)^2)$
3	$1102.50 \times 0.05 = 55.13$	$1157.63 $(1000(1.05)^3)$
4	$1157.63 \times 0.05 = 57.88$	$1215.51 $(1000(1.05)^4)$
5		1276.29
6		1340.10

Equation (Model) $y = 1000(1.05)^x$
which is in general $A = P(1 + i)^n$ (Given on info sheet)

A= Amt present at time t P= Initial Invest i = interest rate n=number of compounding periods over time invested

Note: compounded period here is YEARLY!

Ex) Compute how much money you would have for a $2000 investment compounded

A) <u>annually</u> for 5 years at 4%

Model: Solution:

B) compounded semi-annually for 5 years at 4%

Model: Solution:

C) compounded quarterly for 5 years

Model: Solution:

d) compounded monthly for 5 years

Model: Solution:

e) compounded weekly for 5 years

<u>As you increase the compound periods you increase the amount of interest earned.</u>

f) compounded semi-annually for 6 years at 8% $A= 2000(1+.08/2)^{12}$

Up to this point, students have only been exposed to investments earning compound interest once per year. They should also work with investments that have daily, weekly, monthly, quarterly, semi-annually, or annually compounding periods. Ask students to write an exponential equation if $1000, for example, is invested at 6% compounded monthly. Students should reason that if the interest is compounded every month, annual interest is paid 12 times per year (i.e., the interest rate per compounding period is $i = \frac{\text{annual rate}}{12}$). The compound interest formula is defined as $y = 1000(1.005)^n$ where n is the number of months.

- $3000 was invested at 6% per year compounded monthly. Ask students to answer the following:
  - (i) Write the exponential function in the form $A = P(1 + i)^n$.
  - (ii) Create a table of values of the investment at the end of four months. Use these values to create the equation of the exponential regression function that models the investment.
  - (iii) What do you notice about the equations in (i) and (ii) ?
  - (iv) What will be the future value of the investment after 4 years?

- $2000 is invested at 6% per year compounded semi-annually. Carol defined the exponential function as $A = 2000(1.03)^n$ where n is the number of 6 month periods. Is Carol's reasoning correct? Why or why not?

- Mary needs to borrow $20 000 to buy a car. The bank is charging 7.5% annual interest rate compounded monthly. Her monthly loan payment is $400.76. The time required to pay off the loan can be represented by $(1.00625)^{-n} = 0.6880926$, where n is the number of months. Determine how long it would take her to pay off the loan and how much interest she will have to pay on the loan.

- Joe invests $5000 into a high interest savings bond that has an annual interest rate of 9% compounded monthly. Ask students to answer the following:
  (i) Write the exponential equation $A = P(1 + i)^n$ that represents the situation.
  (ii) How much will the bond be worth after 1 year?

- $1000 is invested at 8.2% per year for 5 years. Using the equation $A = P(1 + i)^n$, ask students to determine the account balance if it is compounded annually, quarterly, monthly and daily. Would it make more of a difference if the interest is accrued for 25 years? Explain your reasoning.

- $2000 is invested for three years that has an annual interest rate of 9% compounded monthly. Lucas solved the following equation $A = 2000(1.0075)^3$. Ask students to identify the error that Lucas made? They should correct the error and solve the problem.
- Ask students to choose between two investment options and justify their choice: earning 12% interest per year compounded annually or 12% interset per year compounded monthly.

use an initial investment of $5000

$$y = 3(1.7)^x$$

increasing curve (Q2 up to Q1)
y-int = 3
domain: xε R
Range: y > 0
no x-int

$$y = 9(.75)^x$$

decreasing curve (x-increases y decreases)
Y-int= 9
domain: xε R
Range Y >0
No x-int